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On Hyperbolic Differential Equation with Periodic Control Initial Condition

Zainab Abdel Ameer[1,*] and Sameer Qasim Hasan[1]

[1] Al-Mustansiriya University, College of Education, Math Dep. Iraq

*Corresponding Author: Zainab Abdel Ameer

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ABSTRACT: The aim of this paper to study the existence solution of some types of hyperbolic differential equation with periodicity of some controls function as nonlocal initial condition for the equation and the technical and used depended on some interest iniquities.

Keywords: Periodic hyperbolic equation, control initial functions, periodic solution

1. INTRODUCTION

In this section we assume the hyperbolic deferential with periodic control initial conditions, as follow:

$$\begin{cases} u_{tt} - \operatorname{div}\left(\sigma\left(|\nabla u|^2\right)\nabla u^m\right) - (\nabla u)^p = |u|^{m-2} f(x,t,u)\log|u|, \quad x\in\Omega, t>0, & (1.1)\\ u(x,t)=0, x\in\partial\Omega, \quad t\in[0,1], & (1.2)\\ u(x,0)=u_0(x), u_t(x,0)=u_1(x), x\in\Omega, & (1.3)\end{cases}$$

$u_0, u_1 \in L^2(\Omega)$, are periodic control functions, Ω is in a bounded domain $\mathbb{R}^n$ with $\partial\Omega$ as smooth bounded $Q_w = R\times(0,T]$, for $[0,1]$. The term $u^{m-2}\log|u|$ is a logarithmic nonlinearity which can be applied to many branches [1–3],. Where $p > 0$ and $m \geq 1$.

Assume the following conditions:

a. $f(x,t,u)$ is Holder continuous in $\bar{\Omega}\times \mathrm{R}\times \mathrm{R}$, periodic in t with a period T and satisfied $f(x,t,u) \leq b_0|u|^{\alpha+1}$ with constant $b_0 \geq 0$ and $0 \leq \alpha \leq 1$.

b. $\sigma\left(|\nabla u|^2\right) = |\nabla u|^m$

c. $\operatorname{div}\left(\sigma\left(|\nabla u|^2\right)\nabla u^m\right)$ is the definition of rather slow.

There are many articles researched on some types of hyperbolic differential equations with some properties and important results such as in[4] the multipoint boundary value problems with nonlocal initial conditions for hyperbolic deferential problems with different approaches. Impulsive System with Periodic Problem for a hyperbolic equation in [5] has been studied extensively. The loaded third order equations as hyperbolic boundary value problems also with mixed types were presented in [6] and in [7] the optimal boundary control problems of the turnpike phenomenon corresponding to hyperbolic systems have been studied and computed. Finely in [8–10] Strict Lyapunov Function used to studied a boundary control of hyperbolic systems.

In this paper, the existence and unique solution of the hyperbolic problem with periodic functions as initial conditions (1.1) - (1.3), and shown that the solution is uniformly bounded, we establish the existence under some conditions are sufficiently conditions to satisfy Leray -Schauder fixed point theorem .

*Corresponding author: zynbaldlal@gmail.com

2. MAIN RESULTS

In this section the results of explain existence of solution for the hyperbolic deferential of problem with initial periodic control functions now, we need the following definition.

Definition (2.1): A function $v \in L^p\left(0, w; w_0^{1,p}(\Omega)\right) \cap C_w\left(\bar{Q}_w\right)$ is called to be a weak solution of the problem (1.1) - (1.3) if $\frac{\partial^2 v}{\partial t^2} \in L^2\left(Q_w\right)$ and v satisfies

$$\iint_{Q_T}\left(\frac{\partial^2 v}{\partial t^2}\varphi + m v^{1-\frac{1}{m}}\sigma\left(|\nabla u|^2\right)\nabla v \nabla \varphi\right) + (m-1)v^{-\frac{1}{m}}\sigma\left(|\nabla u|^2\right)\varphi - (\nabla v)^p \varphi -$$

$|v|^{m-2} f(\alpha, t, v)\log|v|\varphi dxdt\Big) = 0,$ for any $\varphi \in C^2\left(\bar{Q}_w\right)$ with $\varphi(x,0) = u_0(t), \varphi(x,0) = u_1(t)$

And $\varphi \log \Omega \times (0, \tau) = 0$

Theorem(2.1): Let u be solution of $u_{tt} - \operatorname{div}\left(\sigma\left(|\nabla u|^2\right)\nabla u^m\right) - (\nabla u)^p = |u|^{m-2} f(x,t,u)\log|u|$

$u(x,t) = 0, x \in \partial\Omega, t \in [0,1], u(x,0) = u_0(x), u_t(x,0) = u_1(x) \in \Omega$, with $\sigma \in [0,1]$, then there exists positive constant R independent of σ such that $\|u(t)\|_2^2 < R$.

Proof:

$$\frac{1}{(m+2)(m+3)}\frac{\partial^2 u u^{m+3}}{\partial t^2} = \frac{1}{m+2}\left[(m+2)u^{m+1}\left(\frac{\partial u}{\partial t}\right)^2 + u^{m+2}\frac{\partial^2 u}{\partial t^2}\right] = u^{m+1}\nabla u +$$
$$\frac{u^{m+2}}{m+2}\frac{\partial^2 u}{\partial t^2}$$

Thus

$$\frac{1}{m+3}\frac{\partial^2 u}{\partial t^2}u^{m+3} - (m+2)u^{m+1}\nabla u = u^{m+2}\frac{\partial^2 u}{\partial t^2}$$
$$= \frac{1}{m+3}\frac{\partial^2 u}{\partial t^2}u^{m+2} - (m+2)u^{m+1}\nabla u$$

Multiply Eq. (1.1) by u^{m+2}, we get

$$\frac{1}{m+2}\frac{\partial^2 u}{\partial t^2}u^{m+2} - (m+2)u^{m+1}\nabla u - \operatorname{div}\left(\sigma\left(|\nabla u|^2\right)\nabla u^m\right) - \nabla u^m u^{m+2}$$
$$= |u|^{m-2} f(x,t,u)\log|u| u^{m+2}$$

Multiply Eq. (1.1) by $|u(t)|^p u(t), p > 0$, and integrating, we get

$$\int_\Omega \frac{1}{m+2}\frac{\partial^2 u}{\partial t^2}u^{m+2}|u(t)|^p - \int_\Omega (m+2)u^{m+2}\nabla u|u(t)|^p \tag{1.4}$$
$$= \int_\Omega \|\nabla u\big|^2\Big)|u(t)|^p \nabla u^m \nabla u - |u(t)|^p \nabla u^m u^{m+3}\Big\}$$
$$\int_\Omega \frac{1}{m+3}\frac{\partial^2 u}{\partial t^2}u^{m+3}|u(t)|^p - \int_\Omega (m+2)u^{m+2}\nabla u|u(t)|^p +$$
$$\int_\Omega\left\{(p+1)\sigma\left(|\nabla u|^2\right)|u(t)|^p \nabla u^m \nabla u - |u(t)|^p (\nabla u)^p u^{m+3}\right\}$$
$$= \int_\Omega |u|^{m+2} f(x,t,u)|u(t)|^{p+1} u^{m+2}\log|u|$$
$$= \frac{1}{m+2}\frac{\partial^2 u}{\partial t^2}\int_\Omega |\, u(p+$$
$$+ \int_\Omega (p+1)\sigma(t)\Big|^{\frac{m+3}{p}} - \int_\Omega (m+2)\frac{1}{\frac{m+3+p}{p}}\left|\nabla\left(u^{\frac{m+3}{p}}u(t)\right)^p\right|$$
$$\left(|\nabla u|^2\right) m u^{m+2}|\nabla u|^2 - \left|u(t)u^{\frac{m+3}{p}}\nabla u\right|^p$$

$$
\begin{aligned}
&= \int_\Omega \left| u^{\frac{m+2}{p+1}} u(t) u^{\frac{m+2}{p+1}} \right| f(x,t,u) \log|u| \\
&\frac{1}{m+1}\frac{\partial^2}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|^p - \frac{p(m+2)}{m+3+p}\left|\nabla\left(u^{\frac{m+3+p}{p}}\right)^p\right| \\
&+ (p+1)\int_\Omega \left\{ m\sigma\left(|\nabla \mathrm{u}|^2\right)\left|u^{\frac{m-1}{2}}\nabla u\right|^2 - \left|u^{\frac{m+3+p}{p}}\nabla u\right|^p \right\} \\
&= \int_\Omega \left| u^{\frac{2m+5+p}{p+1}} \right|^{p+1} f(x,t,u)\log|u| \\
&\frac{1}{m+1}\frac{\partial^2}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|_p^p - \frac{p(m+2)}{m+3+p}\left|\nabla\left(u^{\frac{m+3+p}{p}}\right)^p\right| \\
&+ (p+1)m\sigma\left(|\nabla \mathrm{u}|^2\right)\frac{2}{m+1}\left\|\nabla u^{\frac{m+1}{2}}\right\|_2^2 - \left\|\frac{p}{m+3+2p}\nabla u^{\frac{m+3+p}{p}}\nabla u\right\|_p^p \\
&= \left\| u^{\frac{2m+5+p}{p+1}} \right\|_{p+1}^{p+1} f(x,t,u)\log|u| \\
&\frac{1}{m+1}\frac{\partial^2}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|_p^p - \frac{p(m+2)}{m+3+p}\|\nabla\left(u^{\frac{m+3+p}{p}}\right)\Big)^p\|_p^p \\
&+ (p+1)\frac{2}{2m+1}\left\|\nabla u^{\frac{2m+1}{2}}\right\|_2^2 - \left\|\frac{p}{m+3+2p}\nabla u^{\frac{m+3+2p}{p}}\right\|_p^p \\
&= \left\| u^{\frac{2m+5+p}{p+1}} \right\|_{p+1}^{p+1} f(x,t,u)\log|u| \\
&\frac{1}{m+1}\frac{\partial^2}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|_p^p - \frac{p(m+2)}{m+3+p}\left\|\nabla u^{\frac{2m+1}{2}}\right\|_p^p + (p+1) \\
&\left(\frac{2m}{2m+1}\left\|\nabla u^{\frac{2m+1}{2}}\right\|_2^2 - \left\|\frac{p}{m+3+2p}\nabla u^{\frac{m+3+2p}{p}}\right\|_p^p\right) \\
&= \left\| u^{\frac{2m+5+p}{p+1}} \right\|_{p+1}^{p+1} f(x,t,u)\log|u|
\end{aligned}
\tag{1.5}
$$

Thus,

$$
\begin{aligned}
&\frac{1}{m+1}\frac{\partial^2 u}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|_p^p - \frac{p(m+2)}{m+3+p}\left\|\nabla\left(u^{\frac{m+3+p}{p}}\right)^p\right\|_p^p + \\
&+ (p+1)\left(\frac{2m}{2m+1}\left\|\nabla u^{\frac{2m+1}{2}}\right\|_2^2 - \left\|\frac{p}{m+3+2p}\nabla u^{\frac{m+3+2p}{p}}\right\|_p^p\right) \\
&\leq \left\| u^{\frac{2m+3+p}{p+1}} \right\|_{p+1}^{p+1} b_0|u|^{\alpha+1}\log|u|
\end{aligned}
$$

Therefore,

$$
\frac{1}{m+1}\frac{\partial^2 u}{\partial t^2}\left\| u^{\frac{m+3+p}{p}} \right\|_p^p - \frac{p(m+2)}{m+3+p}\left\|\nabla\left(u^{\frac{m+3+p}{p}}\right)\right\|_p^p + (p+1)
$$

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$$
\begin{aligned}
&\left(\frac{2m}{2m+1}\left\|\nabla u^{\frac{2m+1}{2}}\right\|_2^2 - \left\|\frac{p}{m+3+2p}\nabla u^{\frac{m+3+2p}{p}}\right\|_p^p\right) \\
&\leq b_0\left\| u^{\frac{2m+6+p+\alpha}{p+1}} \right\|_{p+1}^{p+1}\log|u|
\end{aligned}
$$

Suppose that

$$
\left\|\nabla u^{\frac{2m+1}{2}}\right\|_2^2 < \frac{p(m+2)}{m+3+p}\left\|\nabla\left(u^{\frac{m+3+p}{p}}\right)\right\|_p^p + \frac{(p+1)p}{m+3+2p}\left\|\nabla u^{\frac{m+3+p}{p}}\right\|_p^p, \text{ and}
$$

Set $m = p$, we get that,

$$
\begin{aligned}
&\frac{1}{p+1}\frac{\partial^2}{\partial t^2}\left\|u^{\frac{p+3+p}{p}}\right\|_p^p+(p+1)\frac{2p}{2p+1}\left\|\nabla u^{\frac{2p+1}{2}}\right\|_2^2\\
&=\left\|\nabla u^{\frac{2p+1}{2}}\right\|_2^2\leq b_0\left\|u^{\frac{2p+6+p+\alpha}{p+1}}\right\|_{p+1}^{p+1}\log|u|\\
&\frac{1}{(p+1)}\frac{\partial^2}{\partial t^2}\left\|u^{\frac{2p+3}{4p}}\right\|_p^p+\frac{2(p+1)p}{2p+1}\left\|\nabla u^{\frac{2p+1}{2}}\right\|_2^2-\left\|\nabla u^{\frac{2p+1}{2}}\right\|_2^2\\
&\leq b_0\left\|u^{\frac{3p+6+\alpha}{p+1}}\right\|_{p+1}^{p+1}\log|u|
\end{aligned}
\tag{1.6}
$$

Thus,

$$
\frac{1}{(p+1)}\frac{\partial^2 u}{\partial t^2}\left\|u^{\frac{2p+3}{p}}\right\|_p^p+\left(\frac{2(p+1)p}{2p+1}-1\right)\left\|\nabla u^{\frac{2p+1}{2}}\right\|_2^2\leq b_0\left\|u^{\frac{3p+6+\alpha}{p+1}}\right\|_{p+1}^{p+1}\log|u| \tag{1.7}
$$

Let $u_k=\frac{2p+1}{u^2}$ and $0<\alpha\leq 1$, we have

$$
\frac{1}{(p_k+1)}\frac{\partial^2}{\partial t^2}\|u\|_p^{\frac{2p_k+3}{p_k}}+\left(\frac{2\,(p_k+1)\,p_k}{2p_k+1}-1\right)\|\nabla u_k\|_2^2
$$

$$
\leq b_0\,\|u_k\|_{p_k+1}^{\frac{3p+7}{p+1}}\log|u_k|\,\frac{1}{p_k+1}\frac{\partial}{\partial t^2}\|u_k\|_2^2+\left(\frac{2\,(p_k+1)\,p_k}{2p_k+1}-1\right)
$$

$$
\|\nabla u_k\|_2^2\leq b_0\,\|u_k\|_2^{2\frac{(3p+7)}{p+1}}\log|u_k|\,\frac{\partial}{\partial t^2}\|u_k\|_2^2+\left(\frac{2\,(p_k+1)^2\,p_k-p_k+1}{2p_k+1}\right)
$$

$$
\|\nabla u_k\|_2^2\leq b_0\,(b_k+1)\,\|u_k\|_2^2\,\frac{(3p_k+7)}{p_k+1}\log|u|
$$

$$
\frac{\partial}{\partial t^2}\|u_k\|_2^2\leq\left(1-\frac{2\,(p_k+1)\,p_k}{2p_k+1}\right)(p_k+1)\,\|\nabla u_k\|_2^2+b_0\,(b_k+1)\,\|u_k\|_2^{\frac{2(3p_k+7)}{p_k+1}}\log|u_k| \tag{1.8}
$$

By the Gagliardo-Nirenbery inequality, we have that
$\|u_k(t)\|_2^2\leq C\,\|\nabla u_k(t)\|_2^{\theta}\,\|u_k\big)(t)\|_1^{1-\theta}$, where $\theta=\frac{N}{N+2}\in(0,1)$
Noting that $\|u_k(t)\|_1=\|u_{k-1}(t)\|_2^2$

$$
\frac{\partial^2}{\partial t^2}\|u_k(t)\|_2^2\leq\left(1-\frac{2\,(p_k+1)\,p_k}{2p_k+1}\right)(p_k+1)\,\|\nabla u_k(t)\|_2^{\frac{2}{\theta}}
$$

$$
\|u_k(t)\|_1^{\frac{2(\theta-1)}{\theta}}+b_0\,(b_k+1)\,\|u_k(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}}\log|u_k|
$$

Set $\lambda_k=\max\left\{1,\sup_{\epsilon}\|u_k(t)\|_2^2\right\}$, then
$\frac{\partial^2}{\partial t^2}\|u_k(t)\|_2^2\leq\|u_k(t)\|_2^2$

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$$
\begin{aligned}
&\left\{\left(1-\frac{2\,(p_k+1)\,p_k}{2p_k+1}\right)(p_k+1)\,\|u_k(t)\|_2^{\frac{2}{\theta}-\frac{2(3p_k+7)}{p_k+1}}\lambda_{k-1}^{\frac{4(\theta-1)}{\theta}}+b_0\,(b_k+1)\log|u_k|\right\}\\
&\frac{\partial^2}{\partial t^2}\|u_k(t)\|_2^2\leq\|u_k(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}}\\
&\left\{\left(1-\frac{2\,(p_k+1)\,p_k}{2p_k+1}\right)(p_k+1)\,\|u_k(t)\|_2^{\frac{2}{\theta^-}-\frac{2(3p_k+7)}{p_k+1}}\lambda_{k-1}^{\frac{4(\theta-1)}{\theta}}+b_0\,(b_k+1)\log|u|\right\}\\
&p_{k+1}\frac{\partial^2}{\partial t^2}\|u_k(t)\|_2^{\frac{2}{p_{k+1}}}\leq\left(1-\frac{2\,(p_k+1)\,p_k}{2p_k+1}\right)(p_k+1)\,\|u_k(t)\|_2^{\frac{2}{\theta}-\frac{2(3p_k+7)}{p_k+1}}\lambda_{k-1}^{\frac{4(\theta-1)}{\theta}}\\
&\quad+b_0\,(b_k+1)\log|u_k|
\end{aligned}
\tag{1.9}
$$

Since $u_1(t), u_2(t)$ are periodic control functions.

$$u_k(t)\|_2 \leq \left\{C\left[b_0\left(b_k+1\right)\log|u_k| + \left(\frac{1-2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\ell_{k-1}^{b_k}\right]\lambda_{k-1}^{\frac{4(1-\theta)}{\theta}}\right\}^{\frac{1}{C_k}}$$

Where $C_k = \dfrac{2}{\theta} - \dfrac{2\left(p_k+1\right)}{p_k+2} = \dfrac{2\ell_k}{p_k+2}$

Since $\lambda_{k-1} \geq 1, k = 1, 2,$ we get

$$\|u_k(t)\|_2 \leq \left\{\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\lambda_{k-1}^{b_k+4\frac{(1-\theta)}{\theta}}\right\}^{\frac{1}{C_k}} \tag{1.10}$$

$$= \left\{\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\right\}^{\frac{p_k+2}{2\ell_k}} \quad \frac{4(1-\theta)\left(p_k+2\right)}{2\left(\ell_k-1\right)\theta}$$

$$\|u_{k-1}(t)\|_2 \leq \left\{\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\right\}^{ka'} \lambda_{k-1}^2$$

Where a' is a positive constant independent of k, and $A = 2^{a'}$

$$\ln\|u(t)\|_2 \leq \ln\ell_k \leq \ln\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right) + k\ln A + 2\ln\ell_k - 1$$

Where $A = 2^{a'} > 1$, then

$$\ln\|u(t)\|_2 \leq \ln\ell_k \leq \ln\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)$$

$$\sum_{i=1}^{k-2} 2^1 + 2^{k-1}\ln \mathrm{I}_1 + \ln A\left(\sum_{j=0}^{k-2}(k-j)2^j\right)$$

$\leq \left(2^{k-1}-1\right)\ln\left(1-\frac{2(p_k+1)p_k}{2p_k+1}\right)\left(p_k+1\right) + 2^{k-1}\ln\lambda_1 + f(k)\ln A$, with

$$f(k) = 2^{k+1} - 2^{k-1} - k - 2$$

That is

$$\|u(t)\|_{p_k+2} \leq \left\{\left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\lambda_1^{2^{k-1}}A^{f(k)}\right\}^{\frac{2}{p_k+2}}$$

Letting $k \to \infty$, we get

$$\|u(t)\|_\infty \leq \left(1-\frac{2\left(p_k+1\right)p_k}{2p_k+1}\right)\left(p_k+1\right)\lambda_1^2 \leq \left(\frac{1-2\left(p_k+1\right)p_k}{2p_k+1}\right)(p_k+$$

1. $\left(\max\{1, \sup_t\|u(t)\|_2\}\right)^2$

From (1.7)

$$\frac{\partial}{\partial t^2}\|u(t)\|_2^2 + \left(\frac{2\left(p_k+1\right)^2 p_k}{2p_k+1} - p_k + 1\right)$$

$$\|\nabla u(t)\|_2^2 \leq b_0\left(b_k+1\right)\|u(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}} \log|u_k|$$

According to the Poincare's inequality, we have
$C_p\|u(t)\|_2^2 \leq \|\nabla u\|_2^2$, for sufficiently small $|\Omega|$, we have that

$$\frac{\partial^2}{\partial t^2}\|u(t)\|_2^2 + \left(\frac{2\left(p_k+1\right)^2 p_k}{2p_k+1} - p_k + 1\right)C_p\|u(t)\|^2 \leq b_0\left(b_k+\right.$$

1. $\|u(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}} \log|u|$

Let $\frac{2(p_k+1)p_k}{2p_k+1} < 1$, then

$$\frac{\partial^2}{\partial t^2}\|u(t)\|_2^2 \leq \left(\frac{-2\left(p_k+1\right)p_k}{2p_k+1}+p_k+1\right)\|u(t)\|_2^2 + b_0\left(b_k+1\right)\|u(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}} \log|u_k|$$

$$\frac{\partial^2\|u(t)\|_2^2}{\partial t^2} + \left(p_k+1-\frac{2\left(p_k+1\right)^2 p_k}{2p_k+1}\|u(t)\|_2^2 \leq b_0\left(b_k+1\right)\|u(t)\|_2^{\frac{2(3p_k+7)}{p_k+1}} \log|u_k|\right.$$

$p_k = 0$. To estimate $\|u(t)\|_2^2$ as follows

$$\frac{\partial^2\|u(t)\|_2^2}{\partial t^2} + \|u(t)\|_2^2 \leq b_0\left(b_k+1\right)\|u(t)\|_2^{14} \log|u_k|$$

By young inequality, we obtain
$\frac{\partial^2\|u(t)\|_2^2}{\partial t^2} + \|u(t)\|_2^2 \leq C$
Where C is a constant independent of u. By u_0, u_1 are periodic control function, we get $\|u(t)\|_2^2 \leq R$, where R is a positive constant.

Theorem (2.2): If (a),(b) and (c) hold, then the problem (1.1)-(1.3) admits at least one periodic solution u

Proof First, we define a map by considering the following problem:
$u_{tt} - \operatorname{div}\left(\sigma\left(|\nabla u|^2\right)\nabla u^m\right) - (\nabla u)^p = |u|^{m-2} f(x,t,u)\log|u|, \quad x \in \Omega, t > 0,$
$u(x,t) = 0, x \in \partial\Omega, \quad t \in [0,1]$
$u(x,0) = u_0(x), u_t(x,0) = u_1(x), x \in \Omega,$
Where $f(x,t,u)$ is a given function in $C_T\left(\bar{Q}_T\right)$. If follows from a standard argument similar to

[11] that the problem (1.1)-(1.3) admits a unique solution. So, we define a map $T : C_T\left(\bar{Q}_T\right) \to C_T\left(\bar{Q}_T\right)$. By $u = Tf$ is compact and continuous. In fact, by the method in [15], we can infer that $\|u\|_{L^\infty(Q_T)}$ is bounded if $f \in L^\infty\left(Q_T\right)$ and $u, \nabla u \in C^\infty\left(\bar{Q}_T\right)$. For some $\alpha > 0$. Then (by the ArzelaAscoli theorem) the compactness of the map T comes from $\|u\|_{L^\infty(Q_T)}$ and Holder continuity of u. The continuity of the map T comes from the Holder continuity of ∇u.

3. CONCLUSION:

In conclusion, this paper aimed to study the existence of solutions for certain types of hyperbolic differential equations with periodicity of certain control functions as nonlocal initial conditions. The techniques used in this study were based on analytic dependencies and made use of fixed point theorems to ensure the existence of solutions. This work represents a significant advancement in the field of hyperbolic differential equations and the application of nonlocal initial conditions.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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