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On Degree Topology and Set-T_0 space

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ABSTRACT: This work is aimed to introduce a new topology on a graph, namely the degree topology. This topology is defined by the degree of the vertices of the graphs. We find the degree topology for certain types of graphs and determine their types. The degree topology for the complete graph $K_n$is an indiscrete topology. While The degree topology is generated by a complete bipartite graph $K_{n,m}$ with $n \neq m$ is a quasi-discrete topology. In addition, a new property is initiated namely set-T_0 space and discussed the link between it and T_0 space. We verify that every degree topology is a set-T_0space.

Keywords: Degree- topological Space, Set-T 0 space

1. INTRODUCTION

The field of graph theory is broad and diverse. It is not merely a relational structure; among many other mathematical structures, it can also be seen as topological spaces and combinatorial objects. Graphs can be used to abstractly represent a wide range of notions, which makes them particularly beneficial in real-world applications [1].

Several topological spaces had been built from certain types of graphs. Ahlborn in 1964 established a unique topological space on a digraph $G(V,\ E)$ by any set A in V is openly provided there does not exist an edge from $V - A$ to A [2]. In 2013, Hamza and Faisel constructed a topology on a finite undirected graph on a set of edges and also created a topology on subgraphs. In addition, they investigated the connectedness of these graphs based on the connectedness of their induces topologies of them. They introduced symmetric topological spaces for the isomorphic graphs [3].

Hassan and Abed [4] introduced a family of sub-basis for a topology to define a new independent topology on undirected graphs. In addition, Hassan and Jafar [5] presented a family of sub-basis that motivated a new topology including all vertices non-incident alongside the edge E (non-end vertices of the edge E) on the vertex set V of each simple graph G. Al'Dzhabri et al. initiated DG- topological space by $DG-$open set which is related alongside the digraph $G = (V, E)$ is designated by T_{DG} [6].

This work is aimed to introduce a new topology on a graph, namely the degree topology. This topology is defined by the degree of the vertices of the graphs. We find the degree topology for certain types of graphs and determine their types. The degree topology for the complete graph $K_n$is an indiscrete topology. While The degree topology is generated by a complete bipartite graph $K_{n,m}$ with $n \neq m$ is a quasi-discrete topology. In addition, a new property is initiated namely set-T_0 space and discussed the link between it and T_0 space.

2. PRELIMINARIES

This section includes one of the central concepts which are needed in the rest sections.

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Definition 2.1 [7]

A graph $G = (V, E)$, where V is a set whose elements are called vertices, and E is a set of paired vertices, whose elements are called edges**.**

Definition 2.2 [8] Let X be a set and τ be a collection of subsets of X. If τ has the following properties:

- the empty set and $X$are both in τ
- Any (finite or infinite) union of sets in τ is itself in τ
- Any finite intersection of sets in τ is itself in τ

Then, τ is called a topology on X and the pair (X, τ) is called a topological space.

3. DEGREE-TOPOLOGY

Throughout this section, a new topological space called the degree topology is introduced in the following definition:

Definition 3.1 Let $G(V, E)$ be a simple graph and K be the max degree of all vertices in G. Then, the topology that defines on vertex set $V$and generated by a basis B is called degree topology and it is denoted by T_{deg} where $B_{deg} = (A_i : i = 0, \ldots, K\}$, A_i is the set of all vertices that have a degree i, and K is the maximum degree of all vertices in G.

Example 3.2 Let $G(V, E)$ be a graph as in Figure 3.1 where $V = \{v_1, v_2, v_3, v_4, v_5\}$. Then, $A_0 = \emptyset$, $A_1 = (v_5\}$, $A_2 = (v_1, v_3\}$, $A_3 = (v_2\}$, and $A_4 = (v_4\}$ .So, the basis for T_{deg} is $B_{deg} = \{\emptyset, (v_5\}, (v_1, v_3\}, (v_2\}, (v_4\}$.

Thus, $T_{deg} = \{\emptyset, V, (v_5\}, (v_1, v_3\}, (v_2\}, (v_4\}, (v_5, v_1, v_3\}, (v_5, v_2\}, (v_5, v_4\}, (v_1, v_3, v_2\}, (v_1, v_3, v_4\}, (v_2, v_4\}\}$.

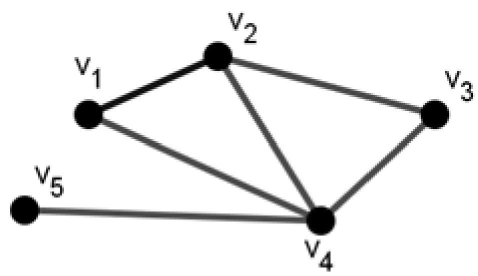

FIGURE 1.

Example 3.3 Let $G(V, E)$be a graph as in Figure 3.2, where $V = (v_1, v_2, v_3, v_4\}$. Then, $A_0 = A_1 = A_2 = \emptyset$, and $A_3 = V$.So, the basis for $T_{deg}$is $B_{deg} = (\emptyset, V\}$. therefore, $T_{deg} = (\emptyset, V\}$.

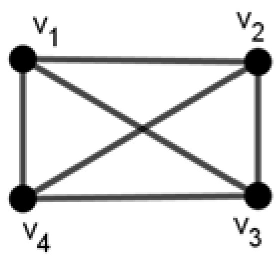

FIGURE 2.

Theorem 3.4 The degree topology generated by a complete bipartite graph $K_{n,m}$ with $n = m$ is an indiscrete topology

Proof:

Assume that$K_{n,m}(V, E)$ is a complete bipartite graph with $n = m$=q. Suppose that the vertex set V partitioned into two disjoint subset V_1 and V_2 such that the number of vertices for V_1 and V_2 is $n$and m, respectively.

By definition of a complete bipartite graph, every vertex in $V_1$is adjacent to all vertices in V_2. Then, the degree of any vertex in $V_1$is$m$ and the degree of any vertex in $V_2$is n. Since $n = m = q$, so the degree of all vertices in $K_{n,m}$ is q. Thus, $A_0 = \emptyset$, and$A_q = V$, hence $B_{deg} = (\emptyset, V\}$. Therefore, $T_{deg} = (\emptyset, V\}$.

Theorem 3.5 The degree topology generated by a complete bipartite graph $K_{n,m}$ with $n \neq m$ is a quasi-discrete topology.

Proof

Assume that $K_{n,m}(V,E)$ a complete bipartite graph with $n \neq m$ and the vertex set V. Let V be partitioned into two disjoint sets V_1 and V_2 and the number of vertices for V_1 and V_2 is $n$and m, respectively.

By definition of the complete bipartite graph, every vertex in $V_1$is adjacent to all vertices in V_2. Then the degree of any vertex in V_1 are m and the degree of any vertex in V_2 is n. Since $n \neq m$, we have $A_0 = \emptyset$,$A_m = V_1$ and$A_n = V_2$. So, the basis for T_{deg} is$(\emptyset, V_1, V_2\}$ and by taking all unions of the degree topology generated by a complete bipartite graph is$(\emptyset, V, V_1, V_2\}$.

Theorem 3.6 The degree topology generated by a complete graph K_n with n vertices is an indiscrete topology.

Proof

Assume that $K_n(V,E)$ is a complete graph with the vertex set $V = \{v_1, v_2, \ldots, v_n\}$. By definition of the complete graph, any two vertices in K_n are adjacent, So, v_1 is adjacent with $v_2, v_3, \ldots, v_n$ so that the degree of $v_1$is $n-1$.

Similarly, v_2 is adjacent with $v_1, v_3, \ldots, v_n$ so that the degree of v_1 is $n-1$and so on. Then, the degree of any vertex in $K_n$has the degree $n-1$, we have $A_0 = \emptyset$, and$A_{n-1} = V$. Thus the basis for T_{deg} is$(\emptyset, V\}$and by taking all unions of the degree- the topology generated by the complete graph is$(\emptyset, V\}$.

Theorem 3.7 The degree topology generated by the cycle graph C_n with n vertices is an indiscrete topology.

Proof:

Assume that $C_n(V,E)$ is a cycle graph with the vertex set $V = \{v_1, v_2, \ldots, v_n\}$. By definition of the cycle graph, it is 2-regular. Then, any vertex in $C_n$has a degree 2, we have $A_0 = \emptyset$, and $A_2 = V$. Thus the basis for T_{deg} is $(\emptyset, V\}$and by taking all unions the degree topology generated by the cycle graph is$(\emptyset, V\}$.

Theorem 3.8 The degree topology generated by the path graph P_n with n vertices is $\{V, \emptyset, (v_1, v_n\}, (v_2, v_3, v_{n-1}\}\}$, where v_1 is the first vertex and v_n is the last vertex.

Proof: Assume that $P_n(V,E)$ is a path graph with the vertex set $V = (v_1, v_2, \ldots, v_n\}$, where v_1 is the first vertex and v_n is the last vertex. By definition of the path graph, the first vertex and the last vertex have degree one while the others vertices have degree 2.

We have $A_0 = \emptyset$, $A_1 = (v_1, v_n\}$ and$A_2 = (v_2, v_3, v_{n-1}\}$, so that the basis for T_{deg} is$(\emptyset, (v_1, v_n\}, (v_2, v_3, v_{n-1}\}\}$and by taking all unions the degree topology generated by the path graph is$(V, \emptyset, (v_1, v_n\}, (v_2, v_3, v_{n-1}\}\}$.

Next, another new concept is initiated, namely the set-T_0space.

4. SET-T_0 SPACE

Definition 4.1 Let X be a non-empty set and τ be a topology on X, then τ is called set-T_0space which is denoted by $T_{0(s)}$ if there exist non-empty sets $M_1, M_2, M_3, \ldots M_K \in \tau$ with $M_i \neq X$ for all $i = 1, 2, \ldots, k$ and k be any natural number such that $\bigcap_{i=1}^{k} M_i = \emptyset$ and, $\bigcup_{i=1}^{k} M_i = X$.

Example 4.2 Let $K_{1,3}(V,E)$ be a simple graph with$V = \{v_1, v_2, v_3, v_4\}$, we have $T_{0(s)} = \{\emptyset, V, (v_1\}, \{v_2, v_3, v_4\}\}$.

Then $T_{0(s)}$ is a set-T_0 space due to existing non-empty open sets $(v_1\}$ and $\{v_2, v_3, v_4\}$ such that $(v_1\} \neq V$, $(v_2, v_3, v_4\} \neq V$, $(v_1\} \cap \{v_2, v_3, v_4\} = \emptyset$ and $(v_1\} \cup (v_2, v_3, v_4\} = X$.

Example 4.3 Let $X = \{y, w, z\}$and $\tau = \{\emptyset, X, (y\}, (z\}, (y, z\}\}$ be a topology on X. Then τ is not a set-T_0space because there are no open subsets of X which satisfy the conditions of a set-T_0 space.

Remark 4.4

1. The set-T_0space is not necessarily satisfied T_0space as in Example 4.2 is a set-T_0space but it is not aT_0space for $v_2 \neq v_1$ with $v_2, v_1 \in V$ and there is no open set containing either $v_1$or v_2, but not both.

2. The T_0space is not necessarily satisfied the set-T_0space as in Example 4.3 which is not a set-T_0space but it is satisfied a T_0-space for $x_2 \neq x_1$ with $x_2, x_1 \in V$ then there exists an open set that contains $x_2$but not x_1 or contains $x_1$but not x_2.

Theorem 4.5 Every degree topology is a set-T_0space/

Proof:

Let $(X, T_{0(s)})$ be a degree- topological space. Since the degree of any vertex is unique. The definition of a degree topology then, the intersection of every non-empty open subset set-T_0 space of the basis for degree topology is empty.

Consequently, we have $\bigcup_{i=1}^{k} M_i = X$, where k is the natural number. Hence $(X, T_{0(s)})$ is a set-T_0 space.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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