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Innovative New Framework of Complex Q-Fuzzy Matrices and Its Application

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ABSTRACT: The complex Q-fuzzy set (CQFS) is an innovative concept that was created by expanding the concept of the complex fuzzy set (CFS) by expanding the universal set X into the Cartesian product between X and the set Q. In this paper, we investigate this idea in a matrix setting by introducing the concept of complex Q-fuzzy matrices (CQFM). We discuss some new elementary operations, including the complement of CQFM, the union, and the intersection between two CQFMs. Moreover, we will show some important algebraic properties that link these processes in a theoretical framework, as well as we familiarize some numerical examples based on CQFM. Then, we promote techniques for determining and adjoins as well as develop methods for computing composition functions to determine the greatest and least eigenvalues of CQFMs. Finally, we improve a multi-step algorithm expending CQFM to represent a problem-solving mechanism in a real-life scenario.

Keywords: Fuzzy set, Q-fuzzy set, complex fuzzy set, complex Q-fuzzy set, medical diagnosis.

1. INTRODUCTION

The uncertainty principle intersects with many everyday problems, posing a challenge for users interested in science, economics, engineering, and medicine. To deal with such scenarios Zadeh [1] introduced a new mathematical tool called fuzzy set (FS) theory. In the mathematical structure of FS, there is a single membership function that acts as a mapping of its domain the universal set, and its codomain, the closed interval [0,1]. Based on this concept, many important mathematical structures have been developed that have proven their flexibility in dealing with the uncertainty inherent in everyday life problems.For example, Mendel [2] researched some types of mathematical relationships between two FSs. Hazaymeh and Bataihah [3,4] presented fixed-point theorems with FSs. With algebraic structures [5,6], Ali et al. [7] introduced a new concept of a PFS under a soft environment.In addition to the many applications of this concept in many different areas of life. Researchers then developed this concept of FS to Q-FS [8] by expanding the universe set to the Cartesian product between the universe set X and the set Q. This expansion provided greater flexibility, as the Q set's components offered a more detailed description of the universal set's components. The role of the FS is evident here by assigning a degree of fuzziness within the closed interval 0 and 1 to each ordered pair resulting from the Cartesian product of the two sets X and Q.This idea has been employed in many studies, for example, Tanamoon et al. [9] introduced Q-fuzzy UP-subalgebras and Q-fuzzy UP-ideals of UP-algebras and investigated their properties. Başer and Uluçay [10] studied the effective properties of Q-FS under an expert system. Adam and Hassan [11] proposed a parameterized soft on Q-FS and applied these tools to problems that contain uncertainties. Al-Sharqi et al. [12] extend the Q-F matrix into a Q-neutrosophic matrix. On the other hand, everything mentioned above regarding Q-FS falls within a single dynamic dimension, i.e., the spatial dimension. In contrast, the research work mentioned above fails to address the complex issues of non-estimation, in which time plays an important role in organising them. To overcome this problem, Ramot et al [13] conceived an innovative concept called complex FS (CFS) as an extension of FS from real value to complex value. Ramot took advantage of the mathematical properties of the unit circle and made it a tool on which the second dimension is conceived. This advanced mathematical technique has gained the approval and satisfaction of many researchers, prompting them to employ this tool in many fields [14-17] using available and innovative mathematical techniques. Yang et al. [18] employed the bipolarity properties on CFS when they proposed that the CF subgroups were under bipolarity properties. Ma et al. [19] introduced some modern operations and laws of CFSs and show its applictions. Zeeshan et al.[20] established some particular examples on CFS and developed a new algorithm using a Cartesian product of CFSs. Hu et al. [21] established the orthogonality relation of CFSs. Al-Qudah et al. [22,23] initiated the definitions of the entropy, similarity distance measures between two CFS under a multi-set effect.

Following on from these works, we will introduce a new algebraic concept called CQ-Fmatrix (CQ-FM) analysis, which utilizes the properties of matrices. These tools are powerful instruments for handling uncertain data by employing matrix properties.

The structure of this article is as follows:To facilitate our discussion, in Part 2, we provide several background definitions regarding CQ-FSM. In Part 3, we will provide a formal definition of a matrix system based on CQ-FM. Following this, we will present the algebraic operations and mathematical properties related to this system as well as we will provide several examples illustrating how these algebraic mathematical tools work. Ultimately, we will use these tools to build a multi-step algorithm to solve one of the everyday decision-making problems.

2. PRELIMINARIES

Definition 2.1[1] A FS Ψ on a reference set $\mathbb{X}$ given as following form:

$$\Psi = \{\langle x, \ddot{\mu}_{\Psi}(x), x \in \mathbb{X}\rangle\}$$

Where the TMD $\ddot{\mu}_{\Psi}(x)$ of element x to $[0,1]$.

Example 2.2 [1] Let $\mathbb{X} = \{x_1, x_2\}$ be non – empty universe set. Then the FS Ψ given as following structure:

$$\Psi = \left\{\frac{\langle 0.1\rangle}{x_1}, \frac{\langle 0.4\rangle}{x_2}\right\}$$

Here we denotes to Ψ is a FS.

Definition 2.3 A Q-FS $\Psi_{\mathbb{Q}}$ on the product between $(\mathbb{X}, \mathbb{Q})$ given as following form:

$$\Psi_{\mathbb{Q}} = \left\{\langle x, \ddot{\mu}_{\Psi_{\mathbb{Q}}}(x, q), x \in \mathbb{X} \; and \; q \in \mathbb{Q}\rangle\right\}$$

Where the TMD $\ddot{\mu}_{\Psi_{\mathbb{Q}}}(x, q)$ of element (x, q) to $[0,1]$.

Example 2.4 Let $\mathbb{X} = \{x_1, x_2\}$ be non – empty universe set and $\mathbb{Q} = \{q\}$. Then the Q-FS $\Psi_{\mathbb{Q}}$ given as following structure:

$$\Psi_{\mathbb{Q}} = \left\{\frac{\langle 0.3\rangle}{(x_1, q)}, \frac{\langle 0.6\rangle}{(x_2, q)}\right\}$$

Here we denotes to $\Psi_{\mathbb{Q}}$ is a Q-FS.

Definition 2.5 Let $\Psi_{\mathbb{Q}_1} = \left\{\left(x, \ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q)\right), x \in \mathbb{X} \; and \; q \in \mathbb{Q}\right\}$ and $\Psi_{\mathbb{Q}_2} = \left\{\left(x, \ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q)\right), x \in \mathbb{X} \; and \; q \in \mathbb{Q}\right\}$ be two Q-FSs on $\mathbb{X}$ then
Then the following properties are done,

i. Complement:

The $\left(\Psi_{\mathbb{Q}_1}\right)^C = \left\{\left(x, \left(\ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q)\right)^C\right), x \in \mathbb{X} \; and \; q \in \mathbb{Q}\right\} = \left\{\left(x, 1 - \ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q)\right), x \in \mathbb{X} \; and \; q \in \mathbb{Q}\right\}$

ii. Union:

The $\Psi_{\mathbb{Q}_1} \cup \Psi_{\mathbb{Q}_2} = \left\{max\left[\ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q), \ddot{\mu}_{\Psi_{\mathbb{Q}_2}}(x, q)\right], x \in \mathbb{X} \; and \; q \in \mathbb{Q}\right\}$.

iii. Intersection:

$\Psi_{\mathbb{Q}_1} \cap \Psi_{\mathbb{Q}_2} = \left\{min\left[\ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q), \ddot{\mu}_{\Psi_{\mathbb{Q}_2}}(x, q)\right], x \in \mathbb{X} \text{ and } q \in \mathbb{Q}\right\}$.

iv. Subset:

The $\Psi_{\mathbb{Q}_1} \subseteq \Psi_{\mathbb{Q}_2}$ if $\ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q) \leq \ddot{\mu}_{\Psi_{\mathbb{Q}_2}}(x, q)$.

v. Equals:

The $\Psi_{\mathbb{Q}_1} = \Psi_{\mathbb{Q}_2}$ if $\ddot{\mu}_{\Psi_{\mathbb{Q}_1}}(x, q) = \ddot{\mu}_{\Psi_{\mathbb{Q}_2}}(x, q)$.

Definition 2.6 A CFS P on a reference set $\mathbb{X}$ given as following form:

$$\mathrm{P} = \{\langle x, \ddot{\mu}_{\mathrm{P}}(x), x \in \mathbb{X} \rangle\}$$

Where $\ddot{\mu}_{\mathrm{P}}(x): \mathbb{X} \longrightarrow \{\ddot{a}: \ddot{a} \in \mathbb{C}, |\ddot{a}| \leq 1\}$ is a complex TM value represented as $\ddot{\mu}_{\mathrm{P}}(x) = \ddot{\rho}_{\mathrm{P}}(x), \mathrm{e}^{i2\pi(\ddot{\varphi}_{\mathrm{P}}(x))}$ such that $\ddot{\rho}_{\mathrm{P}}(x) \in [0,1]$ and $\ddot{\varphi}_{\mathrm{P}}(x) \in [0,2\pi]$.

3. COMPLEX Q-FUZZY MATRIX

In this section of the article, we will provide a formal definition of a matrix system based on CQ-FM. Following this, we will present the algebraic operations and mathematical properties related to this system. Finally, we will provide several examples illustrating how these algebraic mathematical tools work.

Definition 3.1 Consider a reference set $\mathbb{X} = \{x_1, x_2, x_3, \dots, x_n\}$ and a set of $\mathbb{Q} = \{q_1, q_2, q_3, \dots, q_m\}$ such that the $\mathbb{X} \times \mathbb{Q}$ is product between both of them and $A_{\mathbb{Q}}$ be CQ-FS over $\mathbb{X} \times \mathbb{Q}$. Then, the CQ-FS $Y_{\mathbb{Q}}$ given in matrix form as following:

$$A_{\mathbb{Q}_{n\times m}} = \left[a_{\mathbb{Q}_{i\times j}}\right]_{n\times m} = \begin{cases} \left|\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x, q)\right|, & if\,(x, q) \in \mathbb{X} \times \mathbb{Q} \\ (0) & , if\,(x, q) \notin \mathbb{X} \times \mathbb{Q} \end{cases}$$

Where $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x, q) = \ddot{\rho}_{\mathrm{P}_{\mathbb{Q}}}(x, q), \mathrm{e}^{i2\pi\left(\ddot{\varphi}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\right)}$ represents degrees of complex membership of truth on (x, q). Throughout this paper, we will utilize the abbreviation $CQ - FSM_{n\times m}$ for complex Q-fuzzy matrix over $\mathbb{X} \times \mathbb{Q}$.

Example 3.2 Assume that $\mathbb{X} = \{x_1, x_2\}$ represent to two cars under consideration according to the following criteria, represent by $\mathbb{Q} = \{q_1, q_2, q_3\}$ where q_1 = Engine power, q_2 = Color and q_3 = Price. Then the following $Y_{\mathbb{Q}}$ describes the "attractiveness of cars" that is considered for purchase:

$$A_{\mathbb{Q}} = \left\{\left\langle \frac{0.2\mathrm{e}^{2\pi i(0,4)}}{(x_1, q_1)}, \frac{0.5\mathrm{e}^{2\pi i(0,7)}}{(x_1, q_2)}, \frac{0.1\mathrm{e}^{2\pi i(0,2)}}{(x_1, q_3)}, \right.\right.$$

$$\left.\left. \frac{0.6\mathrm{e}^{2\pi i(0,8)}}{(x_2, q_1)}, \frac{0.3\mathrm{e}^{2\pi i(0,9)}}{(x_2, q_2)}, \frac{0.6\mathrm{e}^{2\pi i(0,6)}}{(x_2, q_3)} \right\rangle\right\}$$

Where,

$$0.2\mathrm{e}^{2\pi i(0,4)} = 0.2(\cos(0.8\pi) + i\sin(0.8\pi)) = 0.2(-0.80 + i0.58) = (-0.16 + i0.116)$$

$$|-0.16 + i0.116| = \sqrt{0.0256 + 0.0134 = 0.1974}$$

$$0.5e^{2\pi i(0,7)} = 0.5(\cos(1.4\pi) + i\sin(1.4\pi)) = 0.5(-0.30 - i0.95) = (-0.154 - i0.475)$$

$$|-0.154 - i0.475| = \sqrt{0.0237 + 0.2256 =}\ 0.4993$$

$$0.1e^{2\pi i(0,2)} = 0.1(\cos(0.4\pi) + i\sin(0.4\pi)) = 0.1(0.309 + i0.951) = (-0.154 - i0.475)$$

$$|0.0309 + i0.0951| = \sqrt{0.00095 + 0.00904 =}\ 0.0999$$

$$0.6e^{2\pi i(0,8)} = 0.6(\cos(1.6\pi) + i\sin(1.6\pi)) = 0.6(0.309 - i0.951) = (0.1854 - i0.5706)$$

$$|0.1854 - i0.5706| = \sqrt{0.0343 + 0.3255 =}\ 0.5999$$

$$0.3e^{2\pi i(0,9)} = 0.3(\cos(1.8\pi) + i\sin(1.8\pi)) = 0.3(0.8090 - i0.5877) = (0.2427 - i0.1763)$$

$$|0.2427 - i0.1763| = \sqrt{0.0589 + 0.0310 =}\ 0.2998$$

$$0.6e^{2\pi i(0,6)} = 0.6(\cos(1.2\pi) + i\sin(1.2\pi)) = 0.6(-0.8090 - i0.5877) = (-0.4854 - i0.3526)$$

$$|-0.4854 - i0.3526| = \sqrt{0.2356 + 0.1243 =}\ 0.5999$$

Now, based on the above-mentioned CQFS values, the CQFS is given in matrix form as follows:

$$A_{\mathbb{Q}_{3\times 2}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_1 & 0.1974 & 0.4993 & 0.0999 \\ x_2 & 0.5999 & 0.2998 & 0.5999 \end{bmatrix}$$

Definition 3.3 Assume that $A_{\mathbb{Q}_{n\times m}}$ is CQFM. Then this matrix is called zero CQFM if $\left[a_{\mathbb{Q}_{i\times j}}\right]_{n\times m} = [0]$ for all i and j and this matrix denotes by $0_{\mathbb{Q}_{i\times j}}$.

Definition 3.4 Assume that $A_{\mathbb{Q}_{n\times m}}$ is CQFM. Then this matrix is called universal CQFM if $\left[a_{\mathbb{Q}_{i\times j}}\right]_{n\times m} = [1]$ for all i and j and this matrix denotes by $1_{\mathbb{Q}_{i\times j}}$.

Example 3.5 Let

$$0_{\mathbb{Q}_{3\times 2}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_1 & (0) & 0 & 0 \\ x_2 & 0 & 0 & 0 \end{bmatrix}$$

And

$$1_{\mathbb{Q}_{3\times 2}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_1 & (1) & 1 & 1 \\ x_2 & 1 & 1 & 1 \end{bmatrix}$$

Both matrices above are called the zero CQFM matrix and the universal CQFM matrix, respectively.

Definition 3.6 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then $A_{\mathbb{Q}_{n\times m}}$ is CQF-submatrix of $B_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \sqsubseteq \left[B_{\mathbb{Q}_{n\times m}}\right]$ if $a_{\mathbb{Q}_{i\times j}} \leq b_{\mathbb{Q}_{i\times j}}$ for all i and j.

Definition 3.7 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then $A_{\mathbb{Q}_{n\times m}}$ is CQF-equal matrix of $B_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] = \left[B_{\mathbb{Q}_{n\times m}}\right]$ if $a_{\mathbb{Q}_{i\times j}} = b_{\mathbb{Q}_{i\times j}}$ for all i and j.

Definition 3.8 Assume that $A_{\mathbb{Q}_{n\times m}}$ is CQFM. Then this matrix is called sequel universal CQFM if the number of row (n) equal number if column (m) for all i and j.

Definition 3.9 For a CQFM $A_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the complement of $A_{\mathbb{Q}_{n\times m}}$ is denotes by $\left[A^{C}_{Q_{n\times m}}\right]$ if $c_{\mathbb{Q}_{i\times j}} = 1 - a_{\mathbb{Q}_{i\times j}}$ for all i and j.

Definition 3.10 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the union between $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ is $U_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \sqcup \left[B_{\mathbb{Q}_{n\times m}}\right]$ if $u_{\mathbb{Q}_{i\times j}} = \left[max\left(a_{\mathbb{Q}_{i\times j}}, b_{\mathbb{Q}_{i\times j}}\right)\right]$ for all i and j.

Definition 3.11 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the intersection between $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \sqcap \left[B_{\mathbb{Q}_{n\times m}}\right]$ if $s_{\mathbb{Q}_{i\times j}} = \left[max\left(a_{\mathbb{Q}_{i\times j}}, b_{\mathbb{Q}_{i\times j}}\right)\right]$ for all i and j.

Example 3.12 Assume that the following CQFMs define on product between $\mathbb{X}$ and $\mathbb{Q}$.

$$A_{\mathbb{Q}_{2\times 3}} = \begin{bmatrix} 0.36 & 0.52 & 0.12 \\ 0.48 & 0.73 & 0.59 \end{bmatrix}, B_{\mathbb{Q}_{2\times 3}} = \begin{bmatrix} 0.11 & 0.76 & 0.82 \\ 1 & 0 & 0.63 \end{bmatrix}$$

Then,

$$U_{\mathbb{Q}_{2\times 3}} = \begin{bmatrix} 0.36 & 0.76 & 0.82 \\ 1 & 0.73 & 0.63 \end{bmatrix},$$

$$S_{\mathbb{Q}_{2\times 3}} = \begin{bmatrix} 0.11 & 0.52 & 0.12 \\ 0.48 & 0 & 0.59 \end{bmatrix}$$

And $A^{C}_{Q_{2\times 3}} = \begin{bmatrix} 0.64 & 0.48 & 0.18 \\ 0.52 & 0.27 & 0.41 \end{bmatrix}$

Proposition 3.13 Assume that a CQFM $A_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the following points are verified:

i. $\left(\left(A_{\mathbb{Q}_{n\times m}}\right)^{c}\right)^{c} = A_{\mathbb{Q}_{n\times m}}$.

ii. $\left(\left(0_{\mathbb{Q}_{n\times m}}\right)^{c}\right)^{c} = 1_{\mathbb{Q}_{n\times m}}$.

Proof. follows from definition **3.9**.

Proposition 3.14 For three CQFMs $A_{\mathbb{Q}_{n\times m}}$,$B_{\mathbb{Q}_{n\times m}}$and $D_{\mathbb{Q}_{n\times m}}$define on product between $\mathbb{X}$ and $\mathbb{Q}$.Then the following points are verified:

i. If $A_{\mathbb{Q}_{n\times m}} \leq B_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}} \leq D_{\mathbb{Q}_{n\times m}}$then $A_{\mathbb{Q}_{n\times m}} \leq D_{\mathbb{Q}_{n\times m}}$.

ii. If $A_{\mathbb{Q}_{n\times m}} = B_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}} = D_{\mathbb{Q}_{n\times m}}$then $A_{\mathbb{Q}_{n\times m}} = D_{\mathbb{Q}_{n\times m}}$

Proposition 3.15 For three CQFMs $A_{\mathbb{Q}_{n\times m}}$,$B_{\mathbb{Q}_{n\times m}}$and $D_{\mathbb{Q}_{n\times m}}$define on product between $\mathbb{X}$ and $\mathbb{Q}$.Then the following points are verified:

i. $A_{\mathbb{Q}_{n\times m}} \sqcup B_{\mathbb{Q}_{n\times m}} = B_{\mathbb{Q}_{n\times m}} \sqcup A_{\mathbb{Q}_{n\times m}}$.

ii. $A_{\mathbb{Q}_{n\times m}} \sqcap B_{\mathbb{Q}_{n\times m}} = B_{\mathbb{Q}_{n\times m}} \sqcap A_{\mathbb{Q}_{n\times m}}$.

iii. $\left(A_{\mathbb{Q}_{n\times m}} \sqcup B_{\mathbb{Q}_{n\times m}}\right) \sqcup D_{\mathbb{Q}_{n\times m}} = A_{\mathbb{Q}_{n\times m}} \sqcup \left(B_{\mathbb{Q}_{n\times m}} \sqcup D_{\mathbb{Q}_{n\times m}}\right)$.

iv. $\left(A_{\mathbb{Q}_{n\times m}} \sqcap B_{\mathbb{Q}_{n\times m}}\right) \sqcap D_{\mathbb{Q}_{n\times m}} = A_{\mathbb{Q}_{n\times m}} \sqcap \left(B_{\mathbb{Q}_{n\times m}} \sqcap D_{\mathbb{Q}_{n\times m}}\right)$.

v. $A_{\mathbb{Q}_{n\times m}} \sqcup \left(B_{\mathbb{Q}_{n\times m}} \sqcap D_{\mathbb{Q}_{n\times m}}\right) = \left(A_{\mathbb{Q}_{n\times m}} \sqcup B_{\mathbb{Q}_{n\times m}}\right) \sqcap \left(A_{\mathbb{Q}_{n\times m}} \sqcup D_{\mathbb{Q}_{n\times m}}\right)$.

vi. $A_{\mathbb{Q}_{n\times m}} \sqcap \left(B_{\mathbb{Q}_{n\times m}} \sqcup D_{\mathbb{Q}_{n\times m}}\right) = \left(A_{\mathbb{Q}_{n\times m}} \sqcap B_{\mathbb{Q}_{n\times m}}\right) \sqcup \left(A_{\mathbb{Q}_{n\times m}} \sqcap D_{\mathbb{Q}_{n\times m}}\right)$.

Proof (i). Take $A_{\mathbb{Q}_{n\times m}} \sqcup B_{\mathbb{Q}_{n\times m}} = max\left(a_{\mathbb{Q}_{n\times m}}, b_{\mathbb{Q}_{n\times m}}\right)$

$$= max\left(b_{\mathbb{Q}_{n\times m}}, a_{\mathbb{Q}_{n\times m}}\right) = B_{\mathbb{Q}_{n\times m}} \sqcup A_{\mathbb{Q}_{n\times m}}$$

Proof (ii). Take $A_{\mathbb{Q}_{n\times m}} \sqcap B_{\mathbb{Q}_{n\times m}} = min\left(a_{\mathbb{Q}_{n\times m}}, b_{\mathbb{Q}_{n\times m}}\right)$

$$= min\left(b_{\mathbb{Q}_{n\times m}}, a_{\mathbb{Q}_{n\times m}}\right) = B_{\mathbb{Q}_{n\times m}} \sqcap A_{\mathbb{Q}_{n\times m}}$$

Proof (iii). $\left(A_{\mathbb{Q}_{n\times m}} \sqcup B_{\mathbb{Q}_{n\times m}}\right) \sqcup D_{\mathbb{Q}_{n\times m}} =$

$$= max\left(max\left(A_{\mathbb{Q}_{n\times m}}, B_{\mathbb{Q}_{n\times m}}\right), D_{\mathbb{Q}_{n\times m}}\right) =$$

$$max\left(A_{\mathbb{Q}_{n\times m}}, max\left(B_{\mathbb{Q}_{n\times m}}, D_{\mathbb{Q}_{n\times m}}\right)\right)$$

$$= max\left(A_{\mathbb{Q}_{n\times m}}, \left(B_{\mathbb{Q}_{n\times m}} \sqcup D_{\mathbb{Q}_{n\times m}}\right)\right)$$

$$= A_{\mathbb{Q}_{n\times m}} \sqcup \left(B_{\mathbb{Q}_{n\times m}} \sqcup D_{\mathbb{Q}_{n\times m}}\right).$$

Proof (iv). $\left(A_{\mathbb{Q}_{n\times m}} \sqcap B_{\mathbb{Q}_{n\times m}}\right) \sqcap D_{\mathbb{Q}_{n\times m}} =$

$$= min\left(min\left(A_{\mathbb{Q}_{n\times m}}, B_{\mathbb{Q}_{n\times m}}\right), D_{\mathbb{Q}_{n\times m}}\right) =$$

$$min\left(A_{\mathbb{Q}_{n\times m}}, min\left(B_{\mathbb{Q}_{n\times m}}, D_{\mathbb{Q}_{n\times m}}\right)\right)$$

$$= min\left(A_{\mathbb{Q}_{n\times m}}, \left(B_{\mathbb{Q}_{n\times m}} \sqcap D_{\mathbb{Q}_{n\times m}}\right)\right)$$

$$= A_{\mathbb{Q}_{n\times m}} \sqcap \left(B_{\mathbb{Q}_{n\times m}} \sqcap D_{\mathbb{Q}_{n\times m}}\right).$$

Definition 3.16 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the addition (+) between $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] + \left[B_{\mathbb{Q}_{n\times m}}\right]$ for all i and j and given as following:

$$\left[A_{\mathbb{Q}_{n\times m}}\right] + \left[B_{\mathbb{Q}_{n\times m}}\right] = \left[\frac{a_{\mathbb{Q}_{n\times m}} + b_{\mathbb{Q}_{n\times m}}}{2}\right]$$

Definition 3.17 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the subtraction (-) between $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] - \left[B_{\mathbb{Q}_{n\times m}}\right]$ for all i and j and given as following:

$$\left[A_{\mathbb{Q}_{n\times m}}\right] - \left[B_{\mathbb{Q}_{n\times m}}\right] = \left[\left|a_{\mathbb{Q}_{n\times m}} - b_{\mathbb{Q}_{n\times m}}\right|\right]$$

Definition 3.18 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the multiplication (×) between $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \times \left[B_{\mathbb{Q}_{n\times m}}\right]$ for all i and j and given as following:

$$\left[A_{\mathbb{Q}_{n\times m}}\right] \times \left[B_{\mathbb{Q}_{n\times m}}\right] = \left[\left|a_{\mathbb{Q}_{n\times m}} \times b_{\mathbb{Q}_{n\times m}}\right|\right]$$

Definition 3.19 For CQFM $A_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the scalar multiplication k∈ $[0,1]$ of $A_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left(\mathrm{k}\left[A_{\mathbb{Q}_{n\times m}}\right]\right)$for all i and j and given as following:

$$k\left[A_{\mathbb{Q}_{n\times m}}\right] = \left[\left|ka_{\mathbb{Q}_{n\times m}}\right|\right]$$

Definition 3.20 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the maxim operation operation between $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \vee \left[B_{\mathbb{Q}_{n\times m}}\right]$ for all i and j and given as following:

$$S_{\mathbb{Q}_{n\times m}} = \left[A_{\mathbb{Q}_{n\times m}}\right] \vee \left[B_{\mathbb{Q}_{n\times m}}\right] = \left[max\left(a_{\mathbb{Q}_{n\times m}} \times b_{\mathbb{Q}_{n\times m}}\right)\right]$$

Definition 3.21 For two CQFMs $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ define on product between $\mathbb{X}$ and $\mathbb{Q}$. Then the minimum operation between $A_{\mathbb{Q}_{n\times m}}$ and $B_{\mathbb{Q}_{n\times m}}$ is $S_{\mathbb{Q}_{n\times m}}$ and denotes by $\left[A_{\mathbb{Q}_{n\times m}}\right] \wedge \left[B_{\mathbb{Q}_{n\times m}}\right]$ for all i and j and given as following:

$$S_{\mathbb{Q}_{n\times m}} = \left[A_{\mathbb{Q}_{n\times m}}\right] \wedge \left[B_{\mathbb{Q}_{n\times m}}\right] = \left[min\left(a_{\mathbb{Q}_{n\times m}} \times b_{\mathbb{Q}_{n\times m}}\right)\right]$$

4. APPLICATION IN DECISION-MAKING UNDER THE COMPLEX Q-FUZZY MATRICES

In a car dealership, Mr. Ahmed wants to buy one of three cars, set $\mathbb{X} = \{x_1, x_2, x_3, x_4\}$, and therefore he asks a friendsfor help in evaluating each of these cars according to a number of criteria represented in the set $\mathbb{Q} = \{q_1, q_2, q_3,\}$ where these attributes represent price, car type, and engine power. Therefore, to deal with this issue using the tools presented in this work, we will introduce the following algorithm:

Algorithm:
To implement this performance in accordance with the mathematical structure of our proposed concept, we will introduce the following algorithm.
Step 1. Constructing a CQ-F matrix that expresses the opinion of each of the three experts:
Step 2. Finding the complement of these three matrices.
Step 3. Calculating the maximum value for each matrix according to the **Definition 3.20.**
Step 4. Comparing the values, the highest value represents the best choice.

Now we begin implementing this algorithm by creating three CQ-F matrices, each matrix representing one of the experts.

$$A_{\mathbb{Q}_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_1 & \langle 0.32\rangle & \langle 0.54\rangle & \langle 0.28\rangle \end{bmatrix}$$

$$B_{\mathbb{Q}_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_2 & \langle 0.23\rangle & \langle 0.73\rangle & \langle 0.18\rangle \end{bmatrix}$$

$$C_{\mathbb{Q}_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_3 & \langle 0.19\rangle & \langle 0.59\rangle & \langle 0.72\rangle \end{bmatrix}$$

$$D_{\mathbb{Q}_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_4 & \langle 0.18\rangle & \langle 0.69\rangle & \langle 0.74\rangle \end{bmatrix}$$

$$H_{\mathbb{Q}_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_5 & \langle 0.43\rangle & \langle 0.53\rangle & \langle 0.91\rangle \end{bmatrix}$$

We move to the second step by calculating the complement of these three matrices.

$$A^C_{Q_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_1 & \langle 0.68\rangle & \langle 0.46\rangle & \langle 0.72\rangle \end{bmatrix}$$

$$B^C_{Q_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_2 & \langle 0.77\rangle & \langle 0.27\rangle & \langle 0.82\rangle \end{bmatrix}$$

$$C^{C}_{Q_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_2 & \langle 0.81\rangle & \langle 0.41\rangle & \langle 0.28\rangle \end{bmatrix}$$

$$D^{C}_{Q_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_2 & \langle 0.19\rangle & \langle 0.59\rangle & \langle 0.72\rangle \end{bmatrix}$$

$$H^{C}_{Q_{3\times 1}} = \begin{bmatrix} \mathbb{X}/\mathbb{Q} & q_1 & q_2 & q_3 \\ x_2 & \langle 0.57\rangle & \langle 0.47\rangle & \langle 0.09\rangle \end{bmatrix}$$

After calculating the maximum value for each matrix, we obtain the following values: $x_1 = 72, x_2 = 82 \ \ x_3 = 81, x_4 = 72$and $x_5 = 57$.

Therefore, the x_2 car is the best choice.

In addition, we can illustrate the variation in the severity of the injury among patients through the following statistical chart.

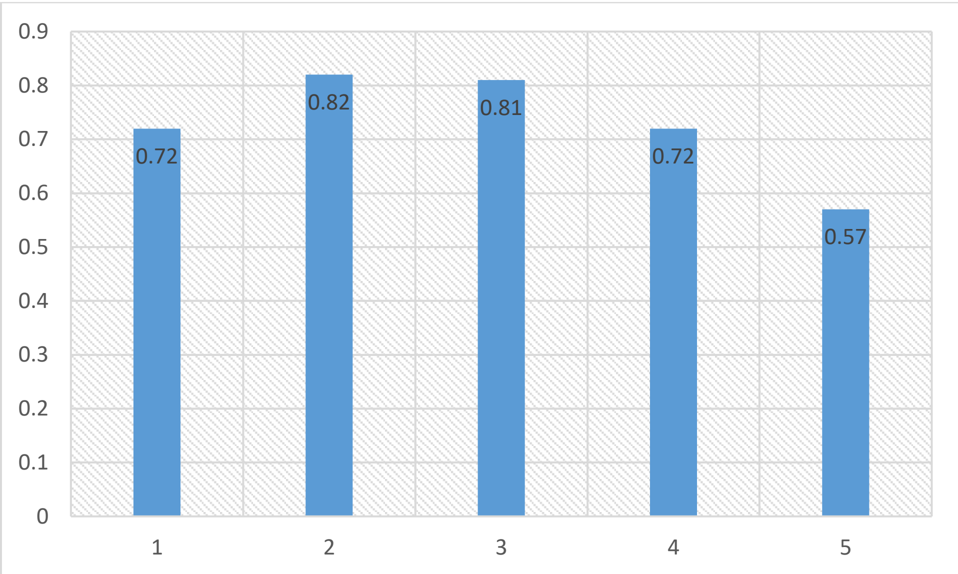

Figure 1: The contrast between injury severity illustrates

5. CONCLUSION

In this paper, we investigated this idea in a matrix setting by introducing the concept of complex Q-fuzzy matrices (CQFM). We discused some new elementary operations, including the complement of CQFM, the union, and the intersection between two CQFMs. Moreover, we showed some important algebraic properties that link these processes in a theoretical framework, as well as we introduced some numerical examples based on CQFM. Then, we promoted techniques for determining and adjoin as well as developed methods for computing composition functions to determine the greatest and least eigenvalues of CQFMs. Finally, we developed a multi-step algorithm using CQFM to represent a problem-solving mechanism in a real-life scenario.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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