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Hybrid Generalization Open Sets in Generalization Topological Spaces

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ABSTRACT: In this paper, we define a new class of generalized open sets in a generalized topological space $(\mathbb{X}, \mathcal{G})$, referred to as $hyg-open$ sets. We further explore topological concepts such as the $hyg-interior, hyg-closure$, and $hyg-exterior$ based on these sets. Additionally, we introduce and examine several types of functions including $hyg-continuous$, $hyg-open$, $hyg-perfectly\ continuous, hyg-contra\ continuous$ function, and $hyg-homeomorphisms$. Various properties and remarks related to these functions are also discussed. Furthermore, we introduce the concepts of GT_{0hyg}, GT_{1hyg} and $GT_{2hyg}-Spaces$, and explore their characterization as well as the relationships among GT_{0hyg}, GT_{1hyg} and $GT_{2hyg}-Spaces$.

Keywords: Generalized topological space GTS, $hyg-open$ sets, $hyg-open$ functions, $hyg-homomorphisms$ and $hyg-$ Separating axioms

1. INTRODUCTION

The concept of open sets plays a vital role in topology and its various applications. Over time, several generalizations of open sets and related functions have been introduced. Levine [6] introduced $semi-open$ sets and $semi-continuous$ functions, while Njastad [7] define $\beta-open$ sets. Biswas [5] explored $semi-open$ function, and Mashhour, Hasanein, and EL-Deeb [1] introduced $\beta-continuous$ and $\beta-open$ mappings. Noiri [10] define totally (perfectly) continuous functions. While Donchev [3] presented $contra-continuous$ functions, and Donchev and Noiri [4] further introduced $contra-semi-continuous$ function. Jafari and Noiri [8] discussed $\beta-contra-continuous$ function. Askander [9] proposed $i-open$ sets along with $i-irresolute$ mappings and $i-homeomorphisms$. Tyagi and Chauhan [2] introduce the notion of $semi-open$ sets and feebly open sets in generalized topological spaces. Császár [11,12,13] examines generalized open sets within the context of generalized topologies. Several separation axioms have been introduced using the framework of generalized open sets, as presented in [14, 15]. Some concepts and notations previously established in [12] are reviewed here, let $\mathbb{X}$ be nonempty set and $\mathcal{G}$ a collection of subsets of $\mathbb{X}$. The collection $\mathcal{G}$ is called a generalized topology (GT) on $\mathbb{X}$ if it contains the empty set $\emptyset$, and the union of any nonempty subcollection of sets in $\mathcal{G}$ also belongs to $\mathcal{G}$. The pair $(\mathbb{X}, \mathcal{G})$ is then referred to as a generalized topological space (GTS) on $\mathbb{X}$. The sets in $\mathcal{G}$ are called $g-open$ sets, and their complements are referred to as $g-closed$ sets [12]. For any subset $\mathbb{A}$ of a generalized topological space $(\mathbb{X}, \mathcal{G})$, the closure and interior of $\mathbb{A}$ with respect to $\mathcal{G}$ are denoted by $cl_g(\mathbb{A})$ and $int_g(\mathbb{A})$, respectively.

This paper is organized into five sections. In the next section, we introduce and investigate a new class of open sets called $hyg-open$ sets. We also define and study topological notions such as $hyg-interior, hyg-closur$, and $hyg-exterior$, all based on the concept of $hyg-open$ sets. In the third section, we present the concepts of $hyg-continuous$ functions, $hyg-open$ functions, $hyg-perfectly\ continuous$ functions, $hyg-contra\ continuous$ functions, and $hyg-homomrphism$. Furthermore, we examine various properties and observations associated with these types of functions. Where in the fourth section, we introduce the notions of GT_{0hyg}, GT_{1hyg} and $GT_{2hyg} - Spaces$. Finely, in the five section a concise conclusion describing all the results obtained in this paper has been given.

2. $hyg - open$ SETS

In this section, we introduce a novel class of open sets referred to as $hyg - open$ sets. Based on this concept, we define and explore several associated notions, including the $hyg - interior$, $hyg - closure$, and $hyg - exterior$. We further investigate the fundamental topological properties of these concepts.

Definitions 2.1. Let $\mathbb{A}$ be a subset of a GTS $(\mathbb{X}, \mathcal{G})$. Then $\mathbb{A}$ is said to be hybrid generalized open set and, denoted by $hyg - open$, if for every non-empty $g - open$ set $G \in \mathcal{G}$, with $G \neq \mathbb{X}$, it holds that $\mathbb{A} \subseteq \left(G \cap cl_g(\mathbb{A})\right)$. The complement of a $hyg - open$ sets is referred to as a $hyg - closed$ sets. The collection of all $hyg - open$ sets in the space $(\mathbb{X}, \mathcal{G})$ is denoted by $hyg - os(\mathbb{X})$

Example 2.2. Let $\mathbb{X} = \{1, 2, 3\}$, $\mathcal{G} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \mathbb{X}\}$.
$hyg - os\ (\mathbb{X}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \mathbb{X}\}$.

Example 2.3. Let $\mathbb{X} = \{a, b, c\}, \mathcal{G} = \{\emptyset, \{a\}, \{a,b\}, \{a,c\}, \mathbb{X}\}$. Then
$hyg - os(\mathbb{X}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \mathbb{X}\}$.

Theorem 2.4. every $g - open$ set is $hyg - open$ set in any $GTS(\mathbb{X}, \mathcal{G})$.

Proof. Let $(\mathbb{X}, \mathcal{G})$ be any topological space and let $\mathbb{U} \subseteq \mathbb{X}$ be any $g - open$ sets. Therefore, $\mathbb{U} \in \mathcal{G}$, $\mathbb{U} \neq \emptyset$. Let us choose $G = \mathbb{U}$. Since $\mathbb{U} \in \mathcal{G}, G \in \mathcal{G}$.Also, assume $\mathbb{U} \neq \mathbb{X}$, $G \neq \mathbb{X}$, and since $\mathbb{U} \neq \emptyset$, we know that $\mathbb{U} \subseteq \mathbb{U} \cap cl_g(\mathbb{U})$,because $\mathbb{U} \subseteq cl_g(\mathbb{U})$. So $\mathbb{U} \subseteq \mathbb{U} \cap cl_g(\mathbb{U})$. By letting $G = \mathbb{U}$, then $\mathbb{U} \subseteq G \cap cl_g(\mathbb{U})$,where $G \in \mathcal{G}, G \neq \emptyset, G \neq \mathbb{X}$. So $\mathbb{U}$ is $hyg - open$ set.■

Remark 2.5. The convers of Theorem 2.4 does not necessarily hold, as evidenced by the following example.

Example 2.6. Let $\mathbb{X} = \{a, b, c\}, \mathcal{G} = \left\{\emptyset, \{a\}, \{a, b\}\right\}$. Then
$hyg - os(\mathbb{X}) = \left\{\emptyset, \{a\}, \{b\}, \{a, b\}\right\}$. The set $\mathbb{A} = \{b\}$ is $hyg - open$, but not $g - open$ set.

Theorem 2.7. Let $(\mathbb{X}, \mathcal{G}\ )$ be a generalized topological space and let $\mathbb{A}, \mathbb{B}$ be a two $hyg - open$ sets. Then

1.$\mathbb{A} \cap \mathbb{B}$ is $hyg - open$ sets.

2.$\mathbb{A} \cup \mathbb{B}$ is $hyg - open$ sets.

Proof. Let $\mathbb{A}, \mathbb{B}$ be a two $hyg - open$ sets, and let $G \subseteq \mathcal{G}$. Then

1. We already have, $\mathbb{A} \subseteq G$, $\mathbb{B} \subseteq G \Rightarrow \mathbb{A} \cap \mathbb{B} \subseteq G$

We also know:
$\mathbb{A} \subseteq cl_g(\mathbb{A})$, $\mathbb{B} \subseteq cl_g(\mathbb{B})$. So $\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{A}) \cap cl_g(\mathbb{B})$. By a basic topological property $cl_g(\mathbb{A} \cap \mathbb{B}) \subseteq cl_g(\mathbb{A}) \cap cl_g(\mathbb{B})$. But this does not imply:

$\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{A} \cap \mathbb{B})$ directly, so we must justify it. However, since $\mathbb{A} \cap \mathbb{B} \subseteq A$, and $\mathbb{A} \subseteq cl_g(\mathbb{A})$, we have:
$\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{A})$. Likewise, $\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{B})$. So, $\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{A}) \cap cl_g(\mathbb{B})$. Therefore:
$\mathbb{A} \cap \mathbb{B} \subseteq G \cap (cl_g(\mathbb{A}) \cap cl_g(\mathbb{B}))$. But $cl_g(\mathbb{A} \cap \mathbb{B} \subseteq cl_g(\mathbb{A}) \cap (\mathbb{B})$. So, $G \cap cl_g(\mathbb{A} \cap \mathbb{B}) \subseteq G \cap (cl_g(\mathbb{A}) \cap cl_g(\mathbb{B}))$

Thus, $\mathbb{A} \cap \mathbb{B} \subseteq G \cap cl_g(\mathbb{A} \cap \mathbb{B})$. Consequently, $\mathbb{A} \cap \mathbb{B}$ is $hyg - open\ \ set$.

2.We have:
$\mathbb{A} \subseteq G$, $\mathbb{B} \subseteq G \Rightarrow \mathbb{A} \cup \mathbb{B} \subseteq G$. Also, $\mathbb{A} \subseteq cl_g(\mathbb{A}), \mathbb{B} \subseteq cl_g(\mathbb{B}) \Rightarrow \mathbb{A} \cup \mathbb{B} \subseteq cl_g(\mathbb{A}) \cup cl_g(\mathbb{B})$. But, $\mathbb{A} \cup \mathbb{B} \subseteq cl_g(\mathbb{A} \cup \mathbb{B})$

Hence, $\mathbb{A} \cup \mathbb{B} \subseteq G \cap cl_g(\mathbb{A} \cup \mathbb{B})$. So $\mathbb{A} \cup \mathbb{B}$ is $hyg - open\ set$.■

Definition 2.8. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space, and let $\mathbb{A} \subseteq \mathbb{X}$. The $hyg - interior$ of $\mathbb{A}$, denoted by $Int_{hyg}(\mathbb{A})$, is define as the union of all $hyg - open$ sets contained in $\mathbb{A}$. By this definition, $Int_{hyg}(\mathbb{A})$ is always an $hyg - open$ set, for every subset $\mathbb{A}$ of $\mathbb{X}$.

Example 2.9. Let $\mathbb{X} = \{a, b, c, d\}, g = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \mathbb{X}\}$ then

$hyg - os(\mathbb{X}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \mathbb{X}\}$

$Int_{hyg}(\{a\}) = \{a\}, Int_{hyg}(\{b\}) = \{b\}, Int_{hyg}(\{c\}) = \{c\}, Int_{hyg}(\{d\}) = \emptyset, Int_{hyg}(\{a,b\}) = \{a,b\}$
$Int_{hyg}(\{a,c\}) = \{a,c\}, Int_{hyg}(\{a,d\}) = \{a\}, Int_{hyg}(\{b,c\}) = \{b,c\}, Int_{hyg}(\{b,d\}) = \{b\}$,
$Int_{hyg}(\{c,d\}) = \{c\}, Int_{hyg}(\{a,b,c\}) = \{a,b,c\}, Int_{hyg}(\{a,b,d\}) = \{a,b\}$,
$Int_{hyg}(\{a,c,d\}) = \{a,c\}, Int_{hyg}(\{b,c,d\}) = \{b,c\}$. Clearly that, if $\mathbb{A}$ is $hyg - open$ then, $\mathbb{A} = Int_{hyg}(\mathbb{A})$.

Proposition 2.10. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space (GTS), and let $\mathbb{A}$ and $\mathbb{B}$ be two subsets of $\mathbb{X}$.Then

1. $\mathbb{A}$ is $hyg - open$ if and only if $\mathbb{A} = Int_{hyg}(\mathbb{A})$.
2. $Int_{hyg}(\mathbb{A}) \subseteq \mathbb{A}$
3. $Int_{hyg}(\mathbb{A}) \subseteq Int_{hyg}(\mathbb{B})$.

Definition 2.11. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space (GTS), and let $\mathbb{A} \subseteq \mathbb{X}$. The $hyg - closure$ of $\mathbb{A}$, denoted by $cl_{hyg}(\mathbb{A})$, is defined as the intersection of all $hyg - closed$ subsets of $\mathbb{X}$ that contain A. By construction, $cl_{hyg}(\mathbb{A})$ is a $hyg - closed$ set for every subset $\mathbb{A}$ of $\mathbb{X}$.

Example 2.12. Let $\mathbb{X} = \{1, 2, 3, 4\}$. $\mathcal{G} = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 3\}, \mathbb{X}\}$. Then

$hyg - os(\mathbb{X}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \mathbb{X}\}$.
$hyg - cs(\mathbb{X}) = \{\emptyset, \{2,3,4\}, \{1,3,4\}, \{1,2,4\}, \{3,4\}, \{2,4\}, \{1,4\}, \{4\}, \mathbb{X}\}$.
$cl_{hyg}(\{1\}) = \{1,4\}, cl_{hyg}(\{2\}) = \{2,4\}, cl_{hyg}(\{3\}) = \{3,4\}, cl_{hyg}(\{4\}) = \{4\}$,
$cl_{hyg}(\{1,2\}) = \{1,2,4\}, cl_{hyg}(\{1,3\}) = \{1,3,4\}, cl_{hyg}(\{1,4\}) = \{1,4\}, cl_{hyg}(\{2,3\}) = \{2,3,4\}$,
$cl_{hyg}(\{2,4\}) = \{2,4\}, cl_{hyg}(\{3,4\}) = \{3,4\}, cl_{hyg}(\{1,2,3\}) = \mathbb{X}, cl_{hyg}(\{1,2,4\}) = \{1,2,4\}$,
$cl_{hyg}(\{1,3,4\}) = \{1,3,4\}, cl_{hyg}(\{2,3,4\}) = \{2,3,4\}$. Clearly that, if $\mathbb{A}$ is $hyg - cs(\mathbb{X})$ then,
$\mathbb{A} = cl_{hyg}(\mathbb{A})$.

Proposition 2.13. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space (GTS) and let $\mathbb{A}$ and $\mathbb{B}$ be two Subsets of $\mathbb{X}$. Then

1. $\mathbb{A}$ is $hyg - cs(\mathbb{X})$ if and only if $\mathbb{A} = Cl_{hyg}(\mathbb{A})$.
2. $\mathbb{A} \subseteq Cl_{hyg}(\mathbb{A})$
3. $Cl_{hyg}(\mathbb{A}) \subseteq Cl_{hyg}(\mathbb{B})$

Lemma 2.14. A subset $\mathbb{A}$ of a generalized topological space $(\mathbb{X}, \mathcal{G})$ is called $hyg - open$ if and only if there exists an open set $\mathbb{U}$ in $\mathbb{X}$ such that $\mathbb{A} \subseteq \mathbb{U} \subseteq cl\,(\mathbb{A})$.
Proof. Uncomplicated. ■

Theorem 2.15. The intersection of a $g - open$ set and a $hyg - open$ set is a $hyg - open$ set .
Proof. Let $\mathbb{A}$ be a $g - open$ set in $\mathbb{X}$, and let $\mathbb{B}$ a $hyg - open$ set in $\mathbb{X}$. Then there exists an open set $\mathbb{U} \subseteq \mathbb{X}$ such that $\mathbb{B} \subseteq \mathbb{U} \subseteq cl(\mathbb{B})$. This implies that $\mathbb{A} \cap \mathbb{B} \subseteq \mathbb{A} \cap \mathbb{U} \subseteq \mathbb{A} \cap cl\,(\mathbb{B}) \subseteq cl(\mathbb{A} \cap \mathbb{B})$.
Since $\mathbb{A} \cap \mathbb{U}$ is generalized open set, it follows from Lemma 2.14 that $\mathbb{A} \cap \mathbb{B}$ is $hyg - os(\mathbb{X})$. ■

Definition 2.16. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space and let $\mathbb{A} \subseteq \mathbb{X}$. Then $ext_{hyg}(\mathbb{A}) = int_{hyg}(\mathbb{X} \backslash \mathbb{A})$ is called the $hyg - exterior$ of A.

Example 2.17. A similar example 2.9.

Theorem 2.18. Let $(\mathbb{X}, \mathcal{G})$ be a generalized topological space (GTS), and let $\mathbb{A}$ and $\mathbb{B}$ be two Subsets of $\mathbb{X}$. Then

1. $ext_{hyg}(\mathbb{A})$ is $hyg - os(\mathbb{X})$ set.
2. If $\mathbb{A} \subseteq \mathbb{B}$, then $ext_{hyg}(\mathbb{B}) \subseteq ext_{hyg}(\mathbb{A})$.
3. $ext_{hyg}\,(\mathbb{A} \cup \mathbb{B}) \subseteq ext_{hyg}(\mathbb{A}) \cap ext_{hyg}(\mathbb{B})$
4. $ext_{hyg}\,(\mathbb{A} \cap \mathbb{B}) \supseteq ext_{hyg}(\mathbb{A}) \cup ext_{hyg}(\mathbb{B})$

Proof. (1) Straightforward.
(2) suppose that $\mathbb{A} \subseteq \mathbb{B}$. Then. $ext_{hyg}(\mathbb{B}) = int_{hyg}(\mathbb{X} \backslash \mathbb{B}) \subseteq int_{hyg}(\mathbb{X} \backslash \mathbb{A}) = ext_{hyg}(\mathbb{A})$
(3) $ext_{hyg}\,(\mathbb{A} \cup \mathbb{B}) = int_{hyg}(\mathbb{X} \backslash (\mathbb{A} \cup \mathbb{B})) = int_{hyg}((\mathbb{X} \backslash \mathbb{A}) \cap (\mathbb{X} \backslash \mathbb{B})) \subseteq int_{hyg}(\mathbb{X} \backslash \mathbb{A}) \cap int_{hyg}(\mathbb{X} \backslash \mathbb{B}) = ext_{hyg}\,(\mathbb{A}) \cap ext_{hyg}(\mathbb{B})$.
(4) $ext_{hyg}(\mathbb{A} \cap \mathbb{B}) = int_{hyg}(\mathbb{X} \backslash (\mathbb{A} \cap \mathbb{B})) = int_{hyg}\big((\mathbb{X} \backslash \mathbb{A}) \cup (\mathbb{X} \backslash \mathbb{B})\big) \supseteq int_{hyg}(\mathbb{X} \backslash \mathbb{A}) \cup int_{hyg}(\mathbb{X} \backslash \mathbb{B}) = ext_{hyg}(\mathbb{A}) \cup ext_{hyg}(\mathbb{B})$. ■

3. $hyg - continuous\ functions$ AND $hyg - homomorphisms$

In this section, we introduce and investigate novel classes of functions, including $hyg - continuous$, $hyg - open$, $hyg - perfectly\ continuous$, $hyg - contra\ continuous$ functions, and $hyg - homomorphisms$, and examine various properties associated with these function.

Definition 3.1. A function $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is said to be $hyg - continuous$, if $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg - open$ set in $\mathbb{X}$ for every $g - open$ set $\mathbb{U}$ in $\mathbb{Y}$.

Example. 3. 2. Let $\mathbb{X} = \{a, b, c\}, \mathbb{Y} = \{1, 2, 3\}$ and, let $\mathcal{G}_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \mathbb{X}\}$, $\mathcal{G}_2 = \{\emptyset, \{1, 2\}, \mathbb{Y}\}$, such that, $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{a\}) = \{1\}$, $\mathcal{F}(\{b\}) = \{3\}$, $\mathcal{F}(\{c\}) = \{2\}$. So, $hyg - os(\mathbb{X}) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \mathbb{X}\}$. It follows directly that the function $\mathcal{F}$ satisfies the condition of $hyg - continuous$.

Theorem 3. 3. Every $g - continuous\ function$ is $hyg - continuous$.

Proof. Let $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ be continuous function and let $\mathbb{U}$ any $g - open$ subset in $\mathbb{Y}$. Since, $\mathcal{F}$ is $g - continuous$, then $\mathcal{F}^{-1}$ $(\mathbb{U})$ is $g - open$ set in $\mathbb{X}$. Since, every $g - open$ set is $hyg - open$ set by Theorem.2. 4, then $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg - open$ set in $\mathbb{X}$. Therefore, $\mathcal{F}$ is $hyg - continuous$ function.■

The converse of the Theorem. 3. 3, need not be true as shown in the following example.

Example. 3. 4. Let $\mathbb{X} = \{b, c, d\}, \mathbb{Y} = \{2, 3, 4\}$ and, let $\mathcal{G}_1 = \{\emptyset, \{b\}, \mathbb{X}\}, \mathcal{G}_2 = \{\emptyset, \{3\}, \{4\}, \mathbb{Y}\}$, such that, $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{b\}) = \{2\}, \mathcal{F}(\{c\}) = \{4\}, f(\{d\}) = \{3\}$. So, $hyg - os\ (\mathbb{X}) = \{\emptyset, \{b\}, \{c\}, \{d\}, \mathbb{X}\}$. It follows directly that the function $\mathcal{F}$ satisfies the condition of $hyg - continuous$, but it is not satisfies $g - continuous$ condition.

Definition 3.5. A function $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is said to be $hyg - open$, if $\mathcal{F}(\mathbb{U})$ is $hyg - open$ set in $\mathbb{Y}$ for every $g - open$ set $\mathbb{U}$ in $\mathbb{X}$.

Example. 3. 6. Let $\mathbb{X} = \mathbb{Y} = \{a, b, c\}$ and, let $\mathcal{G}_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \mathbb{X}\}$, $\mathcal{G}_2 = \{\emptyset, \{a, b\}, \mathbb{Y}\}$, such that, $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{a\}) = \{a\}$, $\mathcal{F}(\{b\}) = \{c\}$, $\mathcal{F}(\{c\}) = \{b\}$. So, $hyg - os(\mathbb{Y}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \mathbb{Y}\}$. Clearly, the function $\mathcal{F}$ is $hyg - open$.

Theorem 3.7. Every $g - open$ function is $hyg - open$.

Proof. Let $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ be $g - open$ function and $\mathbb{U}$ be any $g - open$ set in $\mathbb{X}$. Since, $\mathcal{F}$ is $g - open$, then $\mathcal{F}(\mathbb{U})$ is $g - open$ set in $\mathbb{Y}$. Since, every $g - open$ set is $hyg - open$ set by Theorem.2 .4, then $\mathcal{F}(\mathbb{U})$ is $hyg - open$ set in $\mathbb{Y}$. Therefore, $\mathcal{F}$ is $hyg - open\ function$.

The converse of the above Theorem, need not be true as shown in the following example.

Example. 3.8. in Example 3. 6, the function $f : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is $hyg - open$ but not $g - open$.

Definition 3.9. A bijective function $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is said to be $hyg - homomorphism$ if $\mathcal{F}$ is $hyg - continuous$ and $hyg - open$ function.

Example. 3.10. Let $\mathbb{X} = \{b, c, d\}, \mathbb{Y} = \{2,3,4\}$ and, let $\mathcal{G}_1 = \{\emptyset, \{b\}, \mathbb{X}\}$, $\mathcal{G}_2 = \{\emptyset, \{2\}, \{3\}, \{4\}, Y\}$, such that, $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{b\}) = \{2\}, \mathcal{F}(\{c\}) = \{4\}, \mathcal{F}(\{d\}) = \{3\}$. So, $hyg - os(\mathbb{X}) = \{\emptyset, \{b\}, \{c\}, \{d\}, \mathbb{X}\}$. $hyg - os(Y) = \{\emptyset, \{2\}, \{3\}, \{4\}, Y\}$. Clearly the function $\mathcal{F}$ satisfies the condition of is $hyg - continuous$ and $hyg - open$. So $\mathcal{F}$ is $hyg - homomorphism$.

Theorem 3.11. let $\mathcal{F} : (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is $g -\ homomorphism$, then $\mathcal{F}$ is $hyg - homomorphism$.

Proof. By Theorem 3.3 every $g - continuous$ function is $hyg - continuous$, and by Theorem 3.7, every $g - open$ function is $hyg - open$. Since $\mathcal{F}$ is also bijective, it follows that $\mathcal{F}$ is $hyg -\ homomorphism$.■

The converse of the above Theorem, need not be true as shown in the following example.

Example 3.12 let $\mathbb{X} = \{b, c, d\}, \mathbb{Y} = \{2,3,4\}$ and let $\mathcal{G}_1 = \{\emptyset, \{b\}, \mathbb{X}\}, \mathcal{G}_2 = \{\emptyset, \{2\}, \{3\}, \{4\}, \mathbb{Y}\}$ such that , $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{b\}) = \{2\}, \mathcal{F}(\{c\}) = \{4\}, \mathcal{F}(\{d\}) = \{3\}$. It is clear that, $\mathcal{F}$ is $hyg - homomorphism$ but not $g - homomorphism$.

Definition 3.13. A function $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ is said to be $hyg - perfectly\ continuous$, if $\mathcal{F}^1(\mathbb{U})$ is both $g - open$ and $g - closed$ set in $\mathbb{X}$ for every $hyg - open$ set $\mathbb{U}$ in $\mathbb{Y}$.

Example 3.14. let $\mathbb{X} = \mathbb{Y} = \{a, b, c\}$ and, let $\mathcal{G}_1 = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \mathbb{X}\}, \mathcal{G}_2 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \mathbb{Y}\}$ such that, $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined by $\mathcal{F}(\{a\}) = \{b\}, \mathcal{F}(\{b\}) = \{c\}, \mathcal{F}(\{c\}) = \{a\}. so\ hyg - os(\mathbb{Y}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \mathbb{Y}\}$. It follows directly that the function $\mathcal{F}$ satisfies the condition of $hyg - perfectly\ continuous$.

Theorem 3.15. every $hyg - perfectly\ continuous$ function is $g - perfectly\ ontinuous$

Proof. Let $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ be $hyg - perfectly\ continuous.$ and $\mathbb{U}$ is $hyg - open$ set in $\mathbb{Y}$. Since, every $g - open$ set is $hyg - open$ set by Theorem. 2.4, then $\mathbb{U}$ is $hyg - open$ set in $\mathbb{Y}$. Since, $\mathcal{F}\ hyg - perfectly\ continuous$ function, then $\mathcal{F}^{-1}(\mathbb{U})$ is both $g - open$ and $g - closed$ set in $\mathbb{X}$. Therefore, $\mathcal{F}$ is $hyg - perfectly\ continuous.$ ■

The converse of the above theorem, need not be true as shown in the following example.

Example 3.16. Let $\mathbb{X} = \{a, b, c, d, \}, \mathbb{Y} = \{1,2,3,4\}$ and , let $\mathcal{G}_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, \mathbb{X}\}, \mathcal{G}_2 = \{\emptyset, \{1\}, \{3\}, \{1,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}, \mathbb{Y}\}$, such that , $\mathcal{F}: (\mathbb{X}, \mathcal{G}_1) \to (\mathbb{Y}, \mathcal{G}_2)$ defined $\mathcal{F}(\{a\}) = \{3\}, \mathcal{F}(\{b\}) = \{4\}, \mathcal{F}(\{c\}) = \{1\}, \mathcal{F}(\{d\}) = \{2\}$. So, $hyg - os(\mathbb{Y}) =$

$\{\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,4\},\{2,3,4\},\mathbb{Y}\}$. Clearly the function $\mathcal{F}$ is $g-perfectly\ continuous$ but not $hyg-perfectly\ continuous$.

Definition 3.17. A function $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ is said to be $hyg-contra\ continuous$, if $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg-closed$ set in $\mathbb{X}$ for every $g-open$ set $\mathbb{U}$ in $\mathbb{Y}$.

Example 3.18 let $\mathbb{X}=\mathbb{Y}=\{a,b,c\}$ and , let $\mathcal{G}_1=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},\mathbb{X}\}$, and , let $\mathcal{G}_2=\{\emptyset,\{a\},\{a,b\},\mathbb{Y}$, such that, $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ defined by $\mathcal{F}(\{a\})=\{b\},\mathcal{F}(\{b\})=\{c\},\mathcal{F}(\{c\})=\{a\}$. So, $hyg-os(\mathbb{X})=\{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\mathbb{X}\}$ and, $hyg-cs(\mathbb{X})=\{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\},\mathbb{X}\}$. It follows directly that the function $\mathcal{F}$ satisfies the condition of $hyg-contra\ continuous$.

Theorem 3. 19. Every $g-contra\ continuous$ function is $hyg-contra\ continuous$.

Proof. Let $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ be $g-contra\ continuous$ function and, let $\mathbb{U}$ be any $g-open$ set in $\mathbb{Y}$. Since, $\mathcal{F}$ is $g-contra\ continuous$, then $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg-closed$ set in $\mathbb{X}$. Since, every $g-closed$ set is $hyg-closed$ set, then $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg-closed$ set in $\mathbb{X}$. Therefore, $\mathcal{F}$ is $hyg-contra\ continuous$.

Remark 3. 20. The converse of the Theorem. 3. 19, need not be true as shown in the following example.

Example 3. 21. In example 3. 18, the function $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ is $hyg-contra\ continuous$ but not $g-contra\ continuous$ function.

Theorem 3. 22. Every $g-perfectly\ continuous$ function is $hyg-contra\ continuous$.

Proof. Let $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ be $g-perfectly\ contnuous$ and let $\mathbb{U}$ be any $g-open$ set in $\mathbb{Y}$. Since , $\mathcal{F}$ is $g-perfectly\ continuous$ function the preimage $\mathcal{F}^{-1}(\mathbb{U})$ is both $g-open$ and $g-closed$ in $\mathbb{X}$, and thus it is $g-closed$. This implies that $\mathcal{F}^{-1}(\mathbb{U})$ is $hyg-closed$ in $\mathbb{X}$. Therefore, $\mathcal{F}$ is $hyg-contra\ continuous$ function.■

Remark 3. 23. The convers of the Theorem 3. 22, need not be true as shown in the following example.

Example 3. 24. Let $\mathbb{X}=\mathbb{Y}=\{a,b,c\}$ and, let $\mathcal{G}_1=\{\emptyset,\{a\},\{b\},\{a,b\},\{b,c\},\mathbb{X}\},\mathcal{G}_2=\{\emptyset,\{a\},\{b\},\{a,b\},\mathbb{Y}\}$, such that, $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ defined by $\mathcal{F}(\{a\})=\{b\},\mathcal{F}(\{b\})=\{c\},\mathcal{F}(\{c\})=\{a\}$. So, $hyg-os(\mathbb{X})=\{\emptyset,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\mathbb{X}\}$ and, $hyg-cs(\mathbb{X})=\{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\},\mathbb{X}\}$. Clearly, the function $\mathcal{F}:(\mathbb{X},\mathcal{G}_1)\to(\mathbb{Y},\mathcal{G}_2)$ is $hyg-contra\ continuous$ but not $hyg-perfectly\ continuous$.

4. $hyg-separating\ Axioms$

In this section, we present the notions of GT_{0hyg}, GT_{Ihyg} and $GT_{2hyg-}spaces$, and examine their defining properties along with the interrelationships among them.

Definition 4. 1. A GTS $(\mathbb{X},\mathcal{G})$ is called $GT_{0hyg}-space$ if, for any two distinct points $x,y\in\mathbb{X}$, there exists a $hyg-open$ set $G\in\mathcal{G}$ such that $x\in G$ and $y\notin G$ or vice versa.

Example 4. 2. Let $\mathbb{X}=\{a,b\}$ and define:

$$\mathcal{G}=\{\emptyset,\{a\},\{b\},\mathbb{X}\},\text{ then}$$
$$hyg-os(\mathbb{X})=\{\emptyset,\{a\},\{b\},\mathbb{X}\}$$

for $a\neq b$, the set $\{a\}\in hyg-os(\mathbb{X})$ contains a but not b, so it satisfies GT_{0hyg}.

Theorem 4.3 every $GT_0-space$ is $GT_{ohyg}-space$.

Proof. Let $\mathbb{X}$ be a $GT_0-space$, and let $x_1,x_2\in\mathbb{X}$ be two distinct point. Since $\mathbb{X}$ is a $GT_0-space$, there exists a set $G\in\mathcal{G}-os(\mathbb{X})$ such that, either $x_1\in G$ and $x_2\notin G$, or $x_2\in G$ and $x_1\notin G$. By Theorem 2.4, every $g-open$ set is $hyg-open$ set, it follows that this set G is $hyg-open$ set in $\mathbb{X}$ with the same property, it contains one of the points and excludes the other. Therefore, $\mathbb{X}$ satisfies the condition for being a $GT_{0hyg}-space$. ■

Remark 4.4. in general, the converse of Theorem 4.3 does not hold , as shown in the following example.

Example 4.5. Let $\mathbb{X}=\{12,3\}$ and define:

$$\mathcal{G}=\{\emptyset,\{1,2\},\mathbb{X}\},\text{then}$$

$hyg-os(\mathbb{X})=\{\emptyset,\{1\},\{2\},\{1,2\},\mathbb{X}\}$. Clearly, the $hyg-os(\mathbb{X})$ is $GT_{0hyg}-space$ but, $\mathcal{G}$ is not $GT_0-space$.

definition 4.6. A GTS $(\mathbb{X},\mathcal{G})$ is called $GT_{1hyg}-space$ if, for any two distinct point $x,y\in\mathbb{X}$, there exist a $hyg-open$ sets $Gx,Gy\in\mathcal{G}$, such that $x\in Gx,y\notin Gx$, and $y\in Gy,x\notin Gy$.

Example 4.7. let $\mathbb{X}=\{c,d,e\}$, and let

$$\mathcal{G}=\{\emptyset,,\{c\},\{d,e\},\mathbb{X}\},\text{ then}$$

$hyg-os(\mathbb{X})=\{\emptyset,,\{c\},\{d\},\{e\},\{d,e\},\mathbb{X}\}$. Clearly, the $hyg-os(\mathbb{X})$ is $GT_{1hyg}-space$.

Theorem 4.8. Every $GT_1-space$ is $GT_{1hyg}-space$.

Let $\mathbb{X}$ be a $GT_1-space$, and let x_1 and x_2 be two distinct point in $\mathbb{X}$. Since $\mathbb{X}$ is $GT_1-space$, there exist open sets G and H in $\mathcal{G}$ such that $x_1\in G,x_2\notin G$, and $x_2\in H,x_1\notin H$. Now, by Theorem 2.4, which states that every $\mathcal{G}-open$ set is also a $hyg-open$ set. So, the sets G and H are also $hyg-open$. Therefore, there exist $hyg-open$ sets G and H such that , $x_1\in G,x_2\notin G$ and $x_2\in H$, and $x_1\notin H$. This shows that $\mathbb{X}$ satisfies the separation condition for a $GT_{1hyg}-space$. Hence, $\mathbb{X}$ is a $GT_{1hyg}-space$.

Remark 4.9. In general, the converse of Theorem 4.8 does not hold, as shown in the following example.

Example 4.10. In example 4.7, clear that a $hyg-os(\mathbb{X})$ is $GT_{1hyg}-space$ but,$(\mathbb{X},\mathcal{G})$ not GT_1 .

Definition 4.11. A GTS $(\mathbb{X}, \mathcal{G})$ is called $GT_{2hyg} - space$ if, for any two distinct points $x, y \in \mathbb{X}$,there exists disjoint $hyg - open$ sets $Gx, Gy \in \mathcal{G}$ such that $x \in Gx$, and $y \in Gy$.

Example 4.12. let $\mathbb{X} = \{2, 3, 4, 5\}$, and let:

$$\mathcal{G} = \{\emptyset, \{2\}, \{3\}, \{5\}, \{2,3,5\}, \mathbb{X}\}. \text{ Then}$$

$hyg - os(\mathbb{X}) = \{\emptyset, \{2\}, \{3\}, \{5\}, \{2,3\}, \{2,5\}, \{3,5\}, \{2,3,5\}, \mathbb{X}\}$. Clearly, the $hyg - os(\mathbb{X})$ is $GT_{2hyg-}space$.

Theorem 4. 13. Every $GT_2 - space$ is $GT_{2hyg} - space$.

Proof. Assume that $\mathbb{X}$ be a $GT_{2-}space$, and let x_1 and x_2 be two distinct points in $\mathbb{X}$. Since $\mathbb{X}$ is a $GT_{2-}space$, there exist disjoint $g - open$ sets G and H in $\mathcal{G}$ such that $x_1 \in G$, and $x_2 \in H$. Now, by Theorem 2.4, every $g - open$ set is also a $hyg - open$ set. So, the sets G and H are also disjoint $hyg - open$. Therefore, there exist disjoint $hyg - open$ sets G and H suh that, $x_1 \in G$, and $x_2 \in H$. This shows that $\mathbb{X}$ is a $GT_{2hyg-}space$.■

Remark 4.14. In general, the converse of the above Theorem does not hold, as shown in the following example.

Example 4. 15. Let $\mathbb{X} = \{a, b, c, d\}$, and let:

$$\mathcal{G} = \{\emptyset, \{c\}, \{a, b, d\}, \mathbb{X}\}, \text{ then}$$

$hyg - os\ (\mathbb{X}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \mathbb{X}\}$. Clearly, the $hyg - os\ (\mathbb{X})$ is $GT_{2hyg-}\ space$ but, $(\mathbb{X}, \mathcal{G})$ is not $GT_{2-}space$.

5. CONCLUSION

A new class of sets in generalized topological spaces is introduced and its main properties are studied. The new concept is used to examine generalized continuous functions and some separation axioms. The results indicate this class gives a more refined extension of existing generalized open sets. Therefore, this work strengthens the structure of generalized topology and suggests many future researches related topics.

FUTURE STUDIES

We suggest that upcoming work focus on exploring the relationships between $\boldsymbol{hyg - open}$ sets and various other sets with all applications in $\boldsymbol{GTS}$.

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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