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On Fuzzy Soft $(\alpha,\beta)-class(Q)$ operator

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ABSTRACT: This paper presents a new type of operators, referred to as the fuzzy soft(α,β)-class(Q) operator operator, which operates on a complex Hilbert space. We examine several advantageous properties that are exclusive to this operator class. Additionally, we explore an extended version of several fuzzy soft class(Q) operators in this work, including fuzzy soft(α,β)-class(Q) and fuzzy soft(α,β)-normal operator. We also provide some theorems of operations related to these notions.

Keywords: FS$-class(Q)$ operator, FS(α,β)-normal operator, and fuzzy soft$(\alpha,\beta)-class(Q)$ operator

1. INTRODUCTION

Zadeh in [15] presented important fuzzy set operations and established the concept of fuzzy sets in mathematics. Molodtsov in [13] distinguished a different kind of set called soft sets. and gave important characteristics for this concept in [4]. He further explained that as soft sets relate a collection to a set of parameters, they are mathematical in nature. To attempt to offer a more thorough elucidation of the fuzzy set theory idea Maji [11] introduced a fuzzy soft set (FS) defined as a soft set over the universal set XIs referred to as an FS-set (Fuzzy Soft Set) on X. And various related notions, as a part of a soft set theory [3],[10],. The definition of the FS-Hilbert space was presented by Faried, Ali, and Sakr [5] along with a number of new features and details. Furthermore, among other important ideas, Faried and Ali defined the FS-linear operator and FS-bounded operator within the fuzzy soft Hilbert space [6],[7], and [1],[2]

In recent years, considerable clarity and expansion have been devoted to the class of operators (Q)[9]. This has been achieved by easing certain normality constraints and including new classes, such as (α, β), into the existing class of (Q) operators.

The FS-bounded linear operator $\tilde{I}$ is called a FS-class(Q) operator if $\tilde{\mathfrak{I}}^{*^2}\tilde{\mathfrak{I}}^2 \cong (\tilde{\mathfrak{I}}^*\tilde{\mathfrak{I}})^2$, FS- Normal operator if $\tilde{\mathfrak{I}}^*\tilde{\mathfrak{I}} \cong \tilde{\mathfrak{I}}\tilde{\mathfrak{I}}^*$, and FS$(\alpha,\beta)$-normal operator if $\beta^2\tilde{\mathfrak{I}}^*\tilde{\mathfrak{I}} \gtrapprox \tilde{\mathfrak{I}}\tilde{\mathfrak{I}}^* \gtrapprox \alpha^2\tilde{\mathfrak{I}}^*\tilde{\mathfrak{I}}$ .This work introduces the concept of fuzzy soft (α,β)-class(Q) operators on fuzzy soft Hilbert space $\tilde{H}$, as well as B($\tilde{H}$) denotes the fuzzy soft Banach algebra of all FS-bounded linear operators. which is composed over separable fuzzy soft Hilbert spaces. The paper also presents several theorems that discuss the features of these operators.

2. fuzzy soft (α,β)-class(Q) operator

A new class of fuzzy soft operators on the fuzzy soft operator Hilbert space, called fuzzy soft(α,β)-class(Q) operator, is introduced in this section. We also address several important theorems related to this operator. Initially, the FS class(Q) operators notation was develop.

Definition 2.1 Consider $\tilde{\mathcal{H}}$ to be an FS-Hilbert space , then afuzzy soft bounded linear operator $\tilde{\mathfrak{I}} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$is a FS-$class(Q)$ operator if $\tilde{\mathfrak{I}}^{*^2}\tilde{\mathfrak{I}}^2 \cong (\tilde{\mathfrak{I}}^*\tilde{\mathfrak{I}})^2$,

Theorem 2.2 If $\tilde{\mathfrak{I}} \tilde{\in} FS-class(Q)$ operator, then so is;

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1. $\tilde{e}\tilde{\mathfrak{J}}$ for any FS-real number $\tilde{e}$.
2. For any $\tilde{Q} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$ which is FS- Unitarily Equivalent to $\tilde{\mathfrak{J}}$.

Proof.
1. Suppose $\tilde{\mathfrak{J}} \tilde{\in} FS - class(Q)$, then
$\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \cong (\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2$
$(\tilde{e}\tilde{\mathfrak{J}})^{*^2}(\tilde{e}\tilde{\mathfrak{J}})^2 \cong ((\tilde{e}\tilde{\mathfrak{J}})^*\tilde{e}\tilde{\mathfrak{J}})^2$
After that, we have, $(\tilde{e}^{*^2}\tilde{e}^2)\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \cong (\tilde{e}^{*^2}\tilde{e}^2)(\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2$
Then $\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \cong (\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2$
Therefore, $\tilde{e}\tilde{\mathfrak{J}} \tilde{\in} FS - class(Q)$ operator.
2. Assume that $\tilde{Q} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$. Then there exists an afuzzy soft Unitary operator $\tilde{\mathcal{E}}$ that is if $\tilde{Q} = \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}$ as well $\tilde{Q}^* = \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}$,then:
$\tilde{Q}^{*^2}\tilde{Q}^2 \cong (\tilde{Q}^*\tilde{Q})^2$
$\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{Q}^2 \cong (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2$
$\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\ \tilde{Q}^2 \cong (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2$
$\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}} \cong (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2$, this implies that
$\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2\tilde{\mathcal{E}} \cong (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2$
$\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2\tilde{\mathcal{E}} \cong \tilde{\mathcal{E}}^*(\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2\tilde{\mathcal{E}}$
Hence $\tilde{Q}$ is FS-Unitarily Equivalent to $\tilde{\mathfrak{J}}$.

Proposition 2.3 If $\tilde{\mathfrak{J}} \tilde{\in} FS - class(Q)$ operator ,then $(\tilde{\mathfrak{J}}\tilde{\mathfrak{J}}^*)^2 \cong \tilde{\mathfrak{J}}^2\tilde{\mathfrak{J}}^{*^2}$

Proof: since $\tilde{\mathfrak{J}} \tilde{\in} FS - class(Q)$, and $\tilde{\mathfrak{J}}^* \tilde{\in} FS - class(Q)$, then
$(\tilde{\mathfrak{J}}^{**}\tilde{\mathfrak{J}}^*)^2 \cong (\tilde{\mathfrak{J}}^{**})^2(\tilde{\mathfrak{J}}^*)^2$ this implies that $(\tilde{\mathfrak{J}}\tilde{\mathfrak{J}}^*)^2 \cong \tilde{\mathfrak{J}}^2\tilde{\mathfrak{J}}^{*^2}$.

Now, we will introduce the notation of $FS(\alpha,\beta) - class(Q)$

Definition 2.4 Let $\tilde{\mathcal{H}}$ represent a FS−Hilbert space , then a FS-bounded linear operator $\tilde{\mathfrak{J}} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$is namely fuzzy soft $(\alpha,\beta) - class(Q)$ operator if $\alpha^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \lesssim (\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2 \lesssim \beta^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2$.

Theorem 2.5 If $\tilde{\mathfrak{J}} \tilde{\in} FS - (\alpha,\beta) - class(Q)$ operator, then so is;
1. $\tilde{e}\tilde{\mathfrak{J}}$ for any FS-real number $\tilde{e}$.
2. For any $\tilde{Q} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$ which is fuzzy soft Unitarily Equivalent to $\tilde{\mathfrak{J}}$.

Proof.
1. Suppose $\tilde{\mathfrak{J}} \tilde{\in} FS(\alpha,\beta) - class(Q)$, then
$\alpha^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \lesssim (\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2 \lesssim \beta^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2$
$\alpha^2(\tilde{e}\tilde{\mathfrak{J}})^{*^2}(\tilde{e}\tilde{\mathfrak{J}})^2 \lesssim ((\tilde{e}\tilde{\mathfrak{J}})^*\tilde{e}\tilde{\mathfrak{J}})^2 \lesssim \beta^2(\tilde{e}\tilde{\mathfrak{J}})^{*^2}(\tilde{e}\tilde{\mathfrak{J}})^2$
$\alpha^2(\tilde{e}^{*^2}\tilde{e}^2)\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \lesssim (\tilde{e}^{*^2}\tilde{e}^2)(\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2 \lesssim \beta^2(\tilde{e}^{*^2}\tilde{e}^2)\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2$,
therefore we have
$\alpha^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2 \lesssim (\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}})^2 \lesssim \beta^2\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2$
Hence $\tilde{e}\tilde{\mathfrak{J}} \tilde{\in} FS(\alpha,\beta) - class(Q)$.

2. Assume that $\tilde{Q} \tilde{\in} \mathcal{B}(\tilde{\mathcal{H}})$. Then there exists an afuzzy soft Unitarily operator $\tilde{\mathcal{E}}$, that is if $\tilde{Q} = \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}$ as well $\tilde{Q}^* = \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}$, Then:

$\alpha^2\tilde{Q}^{*^2}\tilde{Q}^2 \lesssim (\tilde{Q}^*\tilde{Q})^2 \lesssim \beta^2\tilde{Q}^{*^2}\tilde{Q}^2$
$\alpha^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{Q}^2 \lesssim (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2 \lesssim \beta^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathcal{E}}\ \tilde{Q}^2$
$\alpha^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\ \tilde{Q}^2 \lesssim (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2 \lesssim \beta^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\ \tilde{Q}^2$
$\alpha^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}} \lesssim (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2 \lesssim \beta^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathcal{E}}\ \tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}}$
$\alpha^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2\tilde{\mathcal{E}} \lesssim (\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^*\tilde{\mathfrak{J}}\tilde{\mathcal{E}})^2 \lesssim \beta^2\tilde{\mathcal{E}}^*\tilde{\mathfrak{J}}^{*^2}\tilde{\mathfrak{J}}^2\tilde{\mathcal{E}}$
Hence, we have $\tilde{Q}$ is fuzzy soft Unitarily Equivalent to $\tilde{\mathfrak{J}}$.

We now show that fuzzy soft (α,β)-class(Q)operator's inverse is as well fuzzy soft (α,β)-class(Q).

Theorem 2.6 If $\widetilde{\mathfrak{I}} \in FS(\alpha,\beta)-class(Q)$ operator, and $\widetilde{\mathfrak{I}}^{-1}$exist, then $\widetilde{\mathfrak{I}}^{-1} \in FS(\alpha,\beta)-class(Q)$.

Proof:
$\alpha^2\widetilde{\mathfrak{I}}^{-1^{*^2}}\widetilde{\mathfrak{I}}^{-1^2} \cong \alpha^2\,\widetilde{\mathfrak{I}}^{*^{-1^2}}\widetilde{\mathfrak{I}}^{-1^2} \cong \alpha^2\widetilde{\mathfrak{I}}^{*^{2^{-1}}}\widetilde{\mathfrak{I}}^{2^{-1}}$then we have
$\cong \alpha^2(\widetilde{\mathfrak{I}}^2\widetilde{\mathfrak{I}}^{*^2})^{-1}$, it implies that $\alpha^2(\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*)^{2^{-1}} \cong \alpha^2(\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*)^{-1^2}$is less than
$\lesseqqgtr (\widetilde{\mathfrak{I}}^{-1^*}\widetilde{\mathfrak{I}}^{-1})^2$, also
$(\widetilde{\mathfrak{I}}^{-1^*}\widetilde{\mathfrak{I}}^{-1})^2 \cong (\widetilde{\mathfrak{I}}^{*^{-1}}\widetilde{\mathfrak{I}}^{-1})^2 \cong (\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*)^{-1^2}$, which implise
$\cong ((\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*)^{2^{-1}} \cong (\widetilde{\mathfrak{I}}^2\widetilde{\mathfrak{I}}^{*^2})^{-1}$, that is
$\lesseqqgtr \beta^2\,\widetilde{\mathfrak{I}}^{*^{2^{-1}}}\widetilde{\mathfrak{I}}^{2^{-1}} \lesseqqgtr \beta^2\,\widetilde{\mathfrak{I}}^{-1^{*^2}}\widetilde{\mathfrak{I}}^{-1^2}$
Therefore, we obtain that $\alpha^2\,\widetilde{\mathfrak{I}}^{-1^{*^2}}\widetilde{\mathfrak{I}}^{-1^2} \lesseqqgtr (\widetilde{\mathfrak{I}}^{-1^*}\widetilde{\mathfrak{I}}^{-1})^2 \lesseqqgtr \beta^2\,\widetilde{\mathfrak{I}}^{-1^{*^2}}\widetilde{\mathfrak{I}}^{-1^2}$.

Proposition 2.7 If $\widetilde{\mathfrak{I}} \in FS(\alpha,\beta)-class(Q)$ operator, such that $\alpha\beta = 1$, then $\widetilde{\mathfrak{I}}^* \in FS(\alpha,\beta)-class(Q)$.

Proof:
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr (\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$
$\alpha^4\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr \alpha^2(\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \alpha^2\beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$ (1)
$\alpha^2\beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr \beta^2(\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \beta^4\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$ (2)
Then from (1) and (2) we have that,
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr \alpha^2\beta^2(\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$
since $\alpha\beta = 1$, then we have
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr (\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$

Theorem 2.8 If $\widetilde{\mathfrak{I}} \in FS(\alpha,\beta)-class(Q)$, and $\tilde{F}$ is FS-self adjoint operator, if $\tilde{F}\widetilde{\mathfrak{I}}$ is commuting isometry, then $\alpha||\tilde{\mathrm{h}}||^2 \lesseqqgtr ||\tilde{F}\widetilde{\mathfrak{I}}||^2 \lesseqqgtr \beta\,||\tilde{\mathrm{h}}||^2$, for all $\tilde{\mathrm{h}} \in \widetilde{\mathcal{H}}$

Proof:
By definition of $\mathrm{FS}(\alpha,\beta)-\mathrm{class}(\mathrm{Q})$ operator , we have that
$\alpha||\tilde{\mathrm{F}}\widetilde{\mathfrak{I}}_{\mathrm{h}}||^2 \lesseqqgtr ||\tilde{\mathrm{F}}^*\,\widetilde{\mathfrak{I}}^*_{\tilde{\mathrm{h}}}||^2 \lesseqqgtr \beta\,||\tilde{\mathrm{F}}\widetilde{\mathfrak{I}}_{\tilde{\mathrm{h}}}||^2$
$\alpha||\tilde{\mathrm{h}}||^2 \lesseqqgtr ||(\widetilde{\mathfrak{I}}\tilde{\mathrm{F}})^*_{\tilde{\mathrm{h}}}||^2 \lesseqqgtr \beta\,||\tilde{\mathrm{h}}||^2$
$\alpha||\tilde{\mathrm{h}}||^2 \lesseqqgtr ||\widetilde{\mathfrak{I}}\tilde{\mathrm{F}}||^2 \lesseqqgtr \beta\,||\tilde{\mathrm{h}}||^2$
$\alpha||\tilde{\mathrm{h}}||^2 \lesseqqgtr ||\tilde{\mathrm{F}}\widetilde{\mathfrak{I}}||^2 \lesseqqgtr \beta\,||\tilde{\mathrm{h}}||^2$

Theorem 2.9 let $\widetilde{\mathfrak{I}} \in \mathrm{FS}(\alpha,\beta)-\mathrm{class}(\mathrm{Q})$ operator, and $\tilde{Q}$ is FS- Unitarily operator, such that $\widetilde{\mathfrak{I}}\tilde{Q} \cong \tilde{Q}\widetilde{\mathfrak{I}}$, then $\widetilde{\mathfrak{I}}\tilde{Q}$ is also $\mathrm{FS}(\alpha,\beta)-\mathrm{class}(\mathrm{Q})$

Proof:
$\alpha^2(\widetilde{\mathfrak{I}}\tilde{Q})^{*^2}(\widetilde{\mathfrak{I}}\tilde{Q})^2 \lesseqqgtr (\widetilde{\mathfrak{I}}^*\tilde{Q}^*\widetilde{\mathfrak{I}}\tilde{Q})^2 \lesseqqgtr \beta^2(\widetilde{\mathfrak{I}}\tilde{Q})^{*^2}(\widetilde{\mathfrak{I}}\tilde{Q})^2$
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\tilde{Q}^{*^2}\widetilde{\mathfrak{I}}^2\tilde{Q}^2 \lesseqqgtr \widetilde{\mathfrak{I}}^*\tilde{Q}^*\widetilde{\mathfrak{I}}^*\tilde{Q}^*\widetilde{\mathfrak{I}}\tilde{Q}\widetilde{\mathfrak{I}}\tilde{Q} \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\tilde{Q}^{*^2}\widetilde{\mathfrak{I}}^2\tilde{Q}^2$,this implies that
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\tilde{Q}^{*^2}\tilde{Q}^2\widetilde{\mathfrak{I}}^2 \lesseqqgtr \widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\tilde{Q}^*\tilde{Q}\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\tilde{Q}^*\tilde{Q} \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\tilde{Q}^{*^2}\tilde{Q}^2\widetilde{\mathfrak{I}}^2$
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr \widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}} \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$, also we obtain
$\alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \lesseqqgtr (\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \lesseqqgtr \beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$
Hence, $\widetilde{\mathfrak{I}}\tilde{Q}$ is $FS(\alpha,\beta)-class(Q)$.

Now, we'll discuss the relationship between $\mathrm{FS}(\alpha,\beta)$-normal operator and $FS(\alpha,\beta)-class(Q)$.

Theorem 2.10 For $\widetilde{\mathfrak{I}} \in \mathcal{B}(\widetilde{\mathcal{H}})$ then every $FS(\alpha,\beta)$-normal operator is $FS(\alpha,\beta)-class(Q)$.

Proof.
Let $\widetilde{\mathfrak{I}}$ *be a* $FS(\alpha,\beta)$-normal, then
$\beta^2\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}} \gtreqqless \widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^* \gtreqqless \alpha^2\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}$ …(1)
The two sides of inequality (1) before and after multiplication
$\beta^2\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}} \gtreqqless \widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}} \gtreqqless \alpha^2\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}}$, and thus
$\beta^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2 \gtreqqless (\widetilde{\mathfrak{I}}^*\widetilde{\mathfrak{I}})^2 \gtreqqless \alpha^2\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2$.

Proposition 2.11 If $\widetilde{\mathfrak{I}}$ is $FS(\alpha,\beta) - class(Q)$ operator, then $\widetilde{\mathfrak{I}}$ is Class (Q), if α,β = 1.

The next two theorems explain more characteristics of the FS$(\alpha,\beta) - class(Q)$ operator**.**

Theorem 2.12 If $\widetilde{\mathfrak{I}}$ and $\tilde{F}$ are commuting two FS$(\alpha,\beta) - class(Q)$ operators, with $\widetilde{\mathfrak{I}}^{*}\tilde{F} = \tilde{F}^{*}\widetilde{\mathfrak{I}}$, then $\widetilde{\mathfrak{I}}\tilde{F}$ is FS$(\alpha,\beta) - class(Q)$ operator

Proof :

$\alpha^2(\widetilde{\mathfrak{I}}\tilde{F})^{*^2}(\widetilde{\mathfrak{I}}\tilde{F})^2 \cong \alpha^2(\tilde{F}^{*^2}\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2\tilde{F}^2) \lneqq \tilde{F}^{*^2}\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2F^2$

$\lneqq (\widetilde{\mathfrak{I}}\tilde{F})^{*^2}(\widetilde{\mathfrak{I}}\tilde{F})^2 \lneqq ((\widetilde{\mathfrak{I}}\tilde{F})^{*}(\widetilde{\mathfrak{I}}\tilde{F}))^2$

and , $((\widetilde{\mathfrak{I}}\tilde{F})^{*}(\widetilde{\mathfrak{I}}\tilde{F}))^2 = \tilde{F}^{*^2}\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2\tilde{F}^2 \lneqq \beta^2(\tilde{F}^{*^2}\widetilde{\mathfrak{I}}^{*^2}\widetilde{\mathfrak{I}}^2\tilde{F}^2) \lneqq \beta^2(\widetilde{\mathfrak{I}}\tilde{F})^{*^2}(\widetilde{\mathfrak{I}}\tilde{F})^2$

Then we have, $\alpha^2(\widetilde{\mathfrak{I}}\tilde{F})^{*^2}(\widetilde{\mathfrak{I}}\tilde{F})^2 \lneqq ((\widetilde{\mathfrak{I}}\tilde{F})^{*}(\widetilde{\mathfrak{I}}\tilde{F}))^2 \lneqq \beta^2(\widetilde{\mathfrak{I}}\tilde{F})^{*^2}(\widetilde{\mathfrak{I}}\tilde{F})^2$

Hence, $\widetilde{\mathfrak{I}}\tilde{F}$ is FS$(\alpha,\beta) - class(Q)$ operator.

Theorem 2.13 If $\widetilde{\mathfrak{I}}$ and $\tilde{\mathrm{A}}$ are commuting two FS$(\alpha,\beta) - class(Q)$ operators, if $\widetilde{\mathfrak{I}}\tilde{\mathrm{T}} \cong \tilde{\mathrm{T}}\widetilde{\mathfrak{I}} \cong \widetilde{\mathfrak{I}}^{*}\tilde{\mathrm{T}}^{*} \cong \tilde{\mathrm{T}}^{*}\widetilde{\mathfrak{I}}^{*} \cong 0$, then $\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}$ is also FS$(\alpha,\beta) - class(Q)$ operator.

Proof :

$\alpha^2(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*^2}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^2 \cong \alpha^2(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})$is equal to

$\cong \alpha^2(\widetilde{\mathfrak{I}}^{*} + \tilde{\mathrm{T}}^{*})(\widetilde{\mathfrak{I}}^{*} + \tilde{\mathrm{T}}^{*})(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})$,through multiplication, we obtain at

$\cong \alpha^2[(\widetilde{\mathfrak{I}}^{*}\widetilde{\mathfrak{I}}^{*} + \widetilde{\mathfrak{I}}^{*}\tilde{\mathrm{T}}^{*} + \tilde{\mathrm{T}}^{*}\widetilde{\mathfrak{I}}^{*} + \tilde{\mathrm{T}}^{*}\tilde{\mathrm{T}}^{*})(\widetilde{\mathfrak{I}}\widetilde{\mathfrak{I}} + \widetilde{\mathfrak{I}}\tilde{\mathrm{T}} + \tilde{\mathrm{T}}\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}\tilde{\mathrm{T}})]$

$\cong \alpha^2(\widetilde{\mathfrak{I}}^{*^2}(\widetilde{\mathfrak{I}}^2 + \tilde{\mathrm{T}}^2) + \tilde{\mathrm{T}}^{*^2}(\widetilde{\mathfrak{I}}^2 + \tilde{\mathrm{T}}^2)$, which implies that

$\cong \alpha^2(\widetilde{\mathfrak{I}}^{*^2} + \tilde{\mathrm{T}}^{*^2})(\widetilde{\mathfrak{I}}^2 + \tilde{\mathrm{T}}^2)$

$\lneqq (\widetilde{\mathfrak{I}}^{*^2} + \tilde{\mathrm{T}}^{*^2})(\widetilde{\mathfrak{I}}^2 + \tilde{\mathrm{T}}^2)$

$\lneqq ((\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}))^2$ … (1)

And thus $((\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}))^2 \cong (\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*^2}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^2$

$\lneqq \beta^2(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*^2}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^2$ … (2)

And thus

$\alpha^2(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*^2}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^2 \lneqq ((\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}))^2 \lneqq \beta^2(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^{*^2}(\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}})^2$

Hence, $\widetilde{\mathfrak{I}} + \tilde{\mathrm{T}}$ is FS$(\alpha,\beta) - class(Q)$ operator.

CONCLUSIONS

The most important results of this study include the following: adding fuzzy soft FS(α,β)-class(Q)operators does not automatically make them FS(α,β)-class(Q) operators on the other hand, adding certain grantee conditions may make them so. Another result is the emergence of several kinds of fuzzy soft operators

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CONFLICTS OF INTEREST

The author declares no conflict of interest.

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