On Tripleα-g-Transformation and Its Properties

Ahmed Mahdi Abbood 1* 1

1Holy Karbala Education Directorate. Holy Karbala. Iraq

*Corresponding Author: Ahmed Mahdi Abbood

ABSTRACT: In this paper, we defined new triple transformation, which is called the fractional triple g-transformation of the order 𝛼, 0 < 𝛼 ≤ 1 for fractional of differentiable functions. This transformation is generalized to double g −transformation. Which has the following form;

\[ T_{g_{\alpha}}(u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_{\alpha}(-(q_1(s)\xi + q_2(s)\tau + q_3(s)\mu)^\alpha) (d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha \]

1. INTRODUCTION

Transforms are powerful tools to solve many engineering and technological problems. It also has wide importance in many fields, including science, engineering, astronomy, and others. The important part of the integrative transformation is the nucleus of this transformation because through the nucleus we can distinguish the type of this transformation. In [1] Jafari introduced a general integral transformation and called it the g transformation. He also studied the properties of this transformation and its applications in differential equations. The mittag-leffler function is an important function and is considered a generalization of the [2],[3] exponential function. In this work, we used the mittag-leffler function as an alternative to the exponential function in g- transform. Also in this paper we studied the properties of the fractional triple g transform and its inverse and its applications in fractional derivatives because most solutions of fractional differential equations are related to the mittag-leffler function. In [4] the triple g transform and its properties were used to solve fractional order partial differential equations based on the Riemann-Liouville fractional derivative. And the caputo fractional derivative, we also presented many theories and examples related to the subject of the paper

2. FRACTIONAL TRIPLE gα-TRANSFORMATION

2.1. DEFINITION [5, 6]:

Let \( u(\xi, \tau, \mu) \) be piecewise function and its continuous where \( \xi, \tau > 0 \) and \( \xi, \tau, \mu \in (0, \infty) \), then the Double g-transformation \( D_{g}(u(\xi, \tau)) \) is defined by the following integral:

\[ D_{g}(u(\xi, \tau)) = p_1 p_2 \int_0^\infty \int_0^\infty e^{-q_1(s)\xi - q_2(s)\tau} u(\xi, \tau) \, dx \, dt \]

such that the integral is convergent for some \( q_1(s), q_2(s) \) are positive functions, and

\[ \|D_{g}(u(\xi, \tau))\| \leq \frac{p_1 p_2 L}{k \xi q_1 q_2}, \quad |u(\xi, \tau)| \leq L e^{k(\xi + \tau)} \]

2.2. DEFINITION [7]:

Let \( u(\xi, \tau, \mu) \) be a function of three variables where \( \xi, \tau, \mu \in [0, \infty) \), then the Triple g-transformation of \( u(\xi, \tau, \mu) \) is defined as;

\[ T_{g_{3}}(u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty e^{-q_1(s)\xi - q_2(s)\tau - q_3(s)\mu} u(\xi, \tau, \mu) \, d\xi \, d\tau \, d\mu \quad , p(s) = p_1 p_2 p_3 \]

*Corresponding author: ahmed2024mahdi@gmail.com

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Such that $\xi, \tau, \mu > 0$ and $s$ is positive constant and $\sup_{e^{2\pi i \theta}} |u(\xi, \tau, \mu)| < 0$, $a, b, c \in R$

The inverse of $T g_3 - \tau$-transform is expressed by the following relationship

$$u(\xi, \tau, \mu) = \frac{1}{2\pi i} \int \frac{1}{\lambda - i\tau} e^{-\xi(\tau - \tau - \mu \xi)} U(s) d\xi$$

### 2.3. DEFINITION [8]:

The $r$-gamma function is given by the formula:

$$\Gamma_{(\omega)} = \frac{1}{\Gamma(\alpha + 1)} \int_0^\infty e^{(\omega-1)^\alpha} E_a(t)\alpha (dt)^\alpha, \text{Re}(\omega) > 0 \quad r > 1$$

In a special case when $\alpha = 1$ we get the classical gamma function $\Gamma_{(\omega)}$

$$\Gamma_{(\omega)} = \int_0^\infty t^{\alpha-1} e^{-t} dt \quad \text{Re}(\omega) > 0$$

### 2.4. DEFINITION:

Let $u(\xi, \tau, \mu)$ be a function of three variables where $\xi, \tau, \mu > 0$, then we define a fractional Triple $g_\alpha$-transformation by the following from:

$$T g_\alpha(u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_a(-\xi, \tau, \mu)(\alpha + 1)^\alpha E_a(-\xi, \tau, \mu)\alpha (dt)^\alpha d\tau d\mu$$

Where $E_a(\omega)$ is mittag-leffler function $E_a(\omega) = \sum_{i=0}^\infty \omega^i \Gamma(\alpha + 1), \text{where} p(s) > 0$ and $q_1(s), q_2(s), q_3(s) > 0$

$$p(s) = p_1p_2p_3$$

### 2.5. REMARK:

The definition (2.4) can be written in another form using the properties of the mittag-leffler function as follow

$$T g_\alpha(u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_a(-\xi, \tau, \mu)\alpha (dt)^\alpha d\tau d\mu$$

### 2.6. PROPOSITION:

If $u_1(\xi, \tau, \mu)$ and $u_2(\xi, \tau, \mu)$ are a functions of three variables $\xi, \tau, \mu$ then

$$T g_\alpha(a_1u_1(\xi, \tau, \mu) + a_2u_2(\xi, \tau, \mu) + a_3u_3(\xi, \tau, \mu)) = a_1T g_\alpha(\xi, \tau, \mu) + a_2T g_\alpha(\xi, \tau, \mu) + a_3T g_\alpha(\xi, \tau, \mu)$$

**Proof:**

The proof is performed from Definition (2.4)

### 2.7. EXAMPLES:

1) $T g_\alpha(1) = p(s) \left[ \left( \int_0^\infty E_a(q_11^\alpha) (1)(d\xi)^\alpha \right) \cdot \left( \int_0^\infty E_a(q_1\tau)^\alpha (d\tau)^\alpha \right) \cdot \left( \int_0^\infty E_a(q_1\mu)^\alpha (d\mu)^\alpha \right) \right]$

$$= p(s) \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha}$$

$$= p(s) \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha}$$

2) $T g_\alpha(\xi^n) = p(s) \left[ \left( \int_0^\infty E_a(q_1\xi)^\alpha (\xi^n)(d\xi)^\alpha \right) \cdot \left( \int_0^\infty E_a(q_1\tau)^\alpha (d\tau)^\alpha \right) \cdot \left( \int_0^\infty E_a(q_1\mu)^\alpha (d\mu)^\alpha \right) \right]$

$$= p(s) \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha} \cdot \frac{\Gamma(1)\Gamma(\alpha + 1)}{q_1^\alpha}$$
\[
\frac{p(s)\Gamma\left(\frac{n}{a} + 1\right)\Gamma^2(1)\Gamma^3(\alpha + 1)}{q_1^{\alpha(n+1)}q_2^a q_3^b}
\]

3) \(T_{\alpha}(\xi^n \tau^m \mu^k) = p(s) \left[ \left( \int_0^\infty E_a(q_1 \xi^a \xi^n) (d\xi)^a \right) \cdot \left( \int_0^\infty E_a(q_1 \tau^a \tau^m) (d\tau)^a \right) \cdot \left( \int_0^\infty E_a(q_1 \mu^a \mu^k) (d\mu)^a \right) \right] \]
\[
= p(s) \left[ \left( \frac{\Gamma\left(\frac{n}{a} + 1\right)\Gamma(\alpha + 1)}{q_1^{\alpha(n+1)}} \right) \cdot \left( \frac{\Gamma\left(\frac{m}{a} + 1\right)\Gamma(\alpha + 1)}{q_2^{\alpha(m+1)}} \right) \cdot \left( \frac{\Gamma\left(\frac{k}{a} + 1\right)\Gamma(\alpha + 1)}{q_3^{\alpha(k+1)}} \right) \right]
\]
\[
= p(s)\Gamma\left(\frac{n}{a} + 1\right)\Gamma\left(\frac{m}{a} + 1\right)\Gamma\left(\frac{k}{a} + 1\right)\Gamma^3(\alpha + 1)
\]

2.8. PROPOSITION:
\(T_{\alpha}(u(a \xi \beta \mu \gamma)) = \frac{p(s)}{a^b c^a d^a} U_{\alpha}\left(\frac{q_1}{a}, \frac{q_2}{b}, \frac{q_3}{c}\right),\) Where \(a, b, c\) are constants. (7)

Proof:
\(T_{\alpha}(u(a \xi \beta \mu \gamma)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_\alpha(-q_1 \xi + q_2 \tau + q_3 \mu) u(a \xi \beta \mu \gamma) (d\xi)^a (d\tau)^a (d\mu)^a\)

let \(\lambda = a \xi, \gamma = \beta \tau, \beta = c \mu\)
\(T_{\alpha}(u(a \xi \beta \mu \gamma)) = \frac{p(s)}{a^b c^a d^a} U_{\alpha}\left(\frac{q_1}{a}, \frac{q_2}{b}, \frac{q_3}{c}\right)\)

2.9. PROPOSITION:
\(T_{\alpha}(E_\alpha(-(a \xi + \beta \tau + c \mu)^a) u(\xi, \tau, \mu)) = p(s) U_{\alpha}(q_1 + a, q_2 + b, q_3 + c)\) (8)

Proof:
\(T_{\alpha}(E_\alpha(-(a \xi + \beta \tau + c \mu)^a) u(\xi, \tau, \mu)) = p \int_0^\infty \int_0^\infty \int_0^\infty E_\alpha(-(a \xi + \beta \tau + c \mu)^a) E_\alpha(-(a \xi + \beta \tau + c \mu)^a) u(\xi, \tau, \mu) (d\xi)^a (d\tau)^a (d\mu)^a\)

By using the formula \(E_\alpha(L\xi + \tau + \mu)^a = E_\alpha(L\xi)^a E_\alpha(L\tau)^a E_\alpha(L\mu)^a\)

Then we have.
\(T_{\alpha}(E_\alpha(-(a \xi + \beta \tau + c \mu)^a) u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_\alpha(-(a \xi + \beta \tau + c \mu)^a) u(\xi, \tau, \mu) (d\xi)^a (d\tau)^a (d\mu)^a\)

2.10. PROPOSITION:
\(T_{\alpha}(\xi^n \tau^m \mu^k u(\xi, \tau, \mu)) = \frac{p(s)}{a^b c^a d^a} T_{\alpha}(u(\xi, \tau, \mu))\) (9)

Proof:
\(T_{\alpha}(\xi^n \tau^m \mu^k u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty \xi^n E_\alpha(-(q_3 \xi)^a) \cdot \xi^m E_\alpha(-(q_3 \xi)^a) \cdot \mu^k E_\alpha(-(q_3 \xi)^a) u(\xi, \tau, \mu) (d\xi)^a (d\tau)^a (d\mu)^a\)

By using the equality \(D_\alpha^n (E_\alpha(-st^n)) = -t^n E_\alpha(-st^n)\).

Then we obtain.
\(T_{\alpha}(\xi^n \tau^m \mu^k u(\xi, \tau, \mu)) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty \frac{\partial}{\partial q_1} E_\alpha(-(q_3 \xi)^a) \cdot \frac{\partial}{\partial q_2} E_\alpha(-(q_3 \xi)^a) \cdot \frac{\partial}{\partial q_3} E_\alpha(-(q_3 \xi)^a) u(\xi, \tau, \mu) (d\xi)^a (d\tau)^a (d\mu)^a\)
\[
\begin{align*}
&= p(s) \int_0^\infty \int_0^\tau \int_0^\infty \frac{\partial^3 \alpha}{\partial q_1^3 \partial q_2^3} E_a(-q_3 \xi)^\alpha E_a(-q_3 \tau)^\alpha u(\xi, \tau, \mu)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha
\end{align*}
\]
Therefor
\[
T_{g_\alpha}(\xi^\alpha \tau^\alpha \mu^\alpha u(\xi, \tau, \mu)) = p(s) \frac{\partial^3 \alpha}{\partial q_1^3 \partial q_2^3} \partial q_3^3 T_{g_\alpha}(u(\xi, \tau, \mu))
\]

2.11. DEFINITION [9]:

Let \( u, v \) are the functions of three variable such that \( \tau, \mu > 0 \), then the fractional triple convolution is defined as follows
\[
(u(\xi, \tau, \mu) ** v(\xi, \tau, \mu)) = \int_0^\tau \int_0^\tau \int_0^\mu u(\xi - \lambda, \tau - \gamma, \mu - \beta) v(\lambda, \gamma, \beta)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha
\]

(10)

2.12. THEOREM:

Let \( u \) and \( v \) be a function then
\[
T_{g_\alpha}(u ** v)(\xi, \tau, \mu) = \frac{1}{p(s)} T_{g_\alpha}(u(\xi, \tau, \mu)). T_{g_\alpha}(v(\xi, \tau, \mu))
\]

(11)

Proof:
\[
T_{g_\alpha}(u ** v)(\xi, \tau, \mu) = p(s) \int_0^\infty \int_0^\tau \int_0^\mu E_a(-q_1 \xi + q_2 \tau + q_3 \mu)^\alpha (u ** v)(\xi, \tau, \mu)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha
\]
\[
= p(s) \int_0^\infty \int_0^\tau \int_0^\mu E_a(-q_1 \tau)^\alpha E_a(-q_3 \mu)^\alpha E_a(-q_3 \tau)^\alpha
\]
\[
\times \left\{ \int_0^\tau \int_0^\mu u(\xi - \lambda, \tau - \gamma, \mu - \beta) v(\lambda, \gamma, \beta)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha \right\}
\]
Let \( n = \xi - \lambda \), \( m = \tau - \gamma \), \( k = \mu - \beta \) and we take limit from 0 to \( \infty \)
\[
= p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_a(-q_1^2 n)^\alpha E_a(-q_2^2 m)^\alpha E_a(-q_3^2 m)^\alpha E_a(-q_1^2 n)^\alpha (u(n, m, k) v(\lambda, \gamma, \beta)(d\xi)^\alpha (d\gamma)^\alpha (d\beta)^\alpha (d\xi)^\alpha (d\gamma)^\alpha (d\beta)^\alpha (d\mu)^\alpha)
\]
\[
= p(s) \int_0^\infty \int_0^\infty \left\{ \int_0^\infty \int_0^\infty \int_0^\infty E_a(-q_1^2 n)^\alpha E_a(-q_2^2 m)^\alpha E_a(-q_1^2 n)^\alpha (u(n, m, k) v(\lambda, \gamma, \beta)(d\xi)^\alpha (d\gamma)^\alpha (d\beta)^\alpha (d\xi)^\alpha (d\gamma)^\alpha (d\beta)^\alpha (d\mu)^\alpha)
\]
Thus
\[
T_{g_\alpha}(u ** v)(x, t, z) = \frac{1}{p(s)} T_{g_\alpha}(u(\xi, \tau, \mu)). T_{g_\alpha}(v(\xi, \tau, \mu))
\]

2.13. DEFINITION:

The fractional delta function of three variables \( \delta_\alpha(\xi - n, \tau - m, \mu - k) \) \( 0 < \alpha \leq 1 \) is defined as follows that.
\[
\int_R \int_R \int_R u(\xi, \tau, \mu) \delta_\alpha(\xi - n, \tau - m, \mu - k)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha = p(s) \alpha^3 u(n, m, k)
\]

(12)

2.14. EXAMPLE:

We taking \( \delta_\alpha(\xi - n, \tau - m, \mu - k) \) then.
\[
T_{g_\alpha}((\xi - n, \tau - m, \mu - k)) = p(s) \int_0^\infty \int_0^\tau \int_0^\mu E_a(-q_1 \xi + q_2 \tau + q_3 \mu)^\alpha \delta_\alpha(\xi - n, \tau - m, \mu - k)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha
\]
\[
= p(s) \alpha^3 E_a(-q_1 \xi + q_2 \tau + q_3 \mu)^\alpha
\]

2.15. THEOREM:

Let \( U_{g_\alpha}(s) \) be the triple \( g_\alpha \) - transform of \( u(\xi, \tau, \mu) \) which define in following formula :
Tgα(u(ξ, τ, μ) = Uαg3(q1q2q3) = p(s) ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(-(q_1(ξ)ξ + q_2(τ)τ + q_3(μ)μ)^α(dξ)^α(dτ)^α(μ)^α

then the inverse formula is defined as;

Tgα⁻¹(Uαg3(s)) = u(ξ, τ, μ)

= 1/(p(s)(m_α)^(3α)) ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(α(q_1(ξ)ξ + q_2(τ)τ + q_3(μ)μ)^α) U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α) (13)

Proof:

u(ξ, τ, μ) = 1/(p(s)(m_α)^(3α)) ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(q_1(ξ)ξ)α E_α(q_2(τ)τ)α E_α(q_3(μ)μ)^α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

× ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(-(q_1(ξ)ξ + q_2(τ)τ + q_3(μ)μ)^α u(λ, γ, β) d(λ)α(dγ)^α(dβ)^α

= 1/(p(s)(m_α)^(3α)) ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(q_1(ξ)ξ)α E_α(q_2(τ)τ)α E_α(q_3(μ)μ)^α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

× ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(-(q_1(ξ)ξ - λ)α E_α(-(q_2(τ)τ - γ)α E_α(-(q_3(μ)μ - β)α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

= 1/(p(s)(m_α)^(3α)) ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(q_1(ξ)ξ)α E_α(q_2(τ)τ)α E_α(q_3(μ)μ)^α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

× ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(q_1(ξ)ξ - λ)α E_α(q_2(τ)τ - γ)α E_α(q_3(μ)μ - β)α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

= 1/(p(s)(m_α)^(3α)) ∫_0^∞ ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(q_1(ξ)ξ)α E_α(q_2(τ)τ)α E_α(q_3(μ)μ)^α U_α(g_3(s)(dq_1)^α(dq_2)^α(dq_3)^α)

= u(ξ, τ, μ)

3. FRACTIONAL TRIPLE g_α -TRANSFORMATION FOR FRACTIONAL PARTIAL DERIVATIVES

In this section we apply the fractional triple transformation to find some partial derivatives. We will explain it through the following theorems;

3.1 THEOREM:

Let u = (ξ, τ, μ) be a function of three variables such that ξ, τ, μ ∈ [0, ∞), α ∈ R^+ then.

Tgα ∂αf(ξ, τ, μ) = q_1αF_a(p, q_1, q_2, q_3) - Γ(1 + α)g_α(fl(0, τ, μ), p_2, q_2) (14)

Proof:

Tgα ∂αf(ξ, τ, μ) = p(s) ∫_0^∞ ∫_0^∞ ∫_0^∞ E_α(-(q_1(ξ)ξ)α E_α(-(q_2(τ)τ)α E_α(-(q_3(μ)μ)^α ∂α/∂ξα f(ξ, τ, μ)(dξ)^α(dτ)^α(μ)^α

= p_1 ∫_0^∞ E_α(-(q_1(ξ)ξ)α ∂α/∂ξα f(ξ, τ, μ)(dξ)^α) p_2 p_3 ∫_0^∞ E_α(-(q_2(τ)τ)α E_α(-(q_3(μ)μ)^α (dτ)^α(μ)^α

By using fractional integration by part formula in the inner integral then we get:

= p_2 p_3 ∫_0^∞ E_α(-(q_2(τ)τ)α E_α(-(q_3(μ)μ)^α [Γ(1 + α)E_α(-(q_1(ξ)ξ)α f(ξ, τ, μ)]_0^∞

= -∫_0^∞ [E_α(-(q_1(ξ)ξ)α ∂α/∂ξα f(ξ, τ, μ)(dξ)^α] (dξ)^α(μ)^α

= -Γ(1 + α)p_2 p_3 ∫_0^∞ E_α(-(q_2(τ)τ)α E_α(-(q_3(μ)μ)^α u(0, τ, μ)(dτ)^α(μ)^α

+ q_1 p_1 p_2 p_3 ∫_0^∞ ∫_0^∞ E_α(-(q_1(ξ)ξ)α E_α(-(q_3(μ)μ)^α u(ξ, τ, μ)(dτ)^α(μ)^α
\[ T_{g_a}(f(\xi, \tau, \mu)) = q_1^2 U_a(p, q_1, q_2, q_3) - \Gamma(1 + \alpha) Dg_a(u(0, \tau, \mu), p_2, q_2) \]

3.2. THEOREM:

Let \( u(\xi, \tau, \mu) \) be a function of three variables such that \( \xi, \tau, \mu \in [0, \infty) \), \( \alpha \in \mathbb{R}^+ \) then.

\[ T_{g_a}(u(\xi, \tau, \mu)) = q_1^2 U_a(p, q_1, q_2, q_3) - \Gamma(1 + \alpha) Dg_a(u(0, \mu), p_1, q_1) \tag{15} \]

**Proof:**

\[
T_{g_a}\left( \frac{\partial^\alpha}{\partial \tau^\alpha} f(\xi, \tau, \mu) \right) = p(s) \int_0^\infty \int_0^\infty \int_0^\infty E_a(-q_1^2 \tau^\alpha) \frac{\partial^\alpha}{\partial \tau^\alpha} u(\xi, \tau, \mu)(d\xi)^{\alpha}(d\mu)^{\alpha}
\]

By using fractional integration by part formula in the inner integral then we get:

\[
= p_1 p_2 \int_0^\infty \int_0^\infty E_a(-q_1^2 \tau^\alpha) \frac{\partial^\alpha}{\partial \tau^\alpha} u(\xi, \tau, \mu)(d\xi)^{\alpha}(d\mu)^{\alpha}
\]

3.3. THEOREM:

Let \( u(\xi, \tau, \mu) \) be a function such that \( \xi, \tau, \mu \in [0, \infty) \), \( \alpha \in \mathbb{R}^+ \) then

\[ T_{g_a}(u(\xi, \tau, \mu)) = (\alpha!)^2 u(0, 0, \mu) - (\alpha!) q_2^2 U_a(0, q_2, q_3) - (\alpha!) q_1^2 U_a(q_1, 0, q_3) + q_1^2 q_2^2 q_3^2 U_a(\xi, \tau, \mu) \]

**Proof:**

The proof is complete by using the Theorem (3.1) and Theorem (3.2)

4. CONCLUSION

In this article, we have covered a new definition of the fractional triple g-transform and its inverse. As we discussed some of the characteristics of this transformation, while studying some theorems and examples. In addition, we found the fractional triple transformation for fractional partial order derivatives.

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