WJCMS

Wasit Journal of Computer and Mathematics Science

Journal Homepage: https://wjcm.uowasit.edu.iq/index.php/WJCM

e-ISSN: 2788-5879 p-ISSN: 2788-5879

Topological Mappings Based on SPG*-Closed

Marwa Munther Hassan* **and Prof. Dr. Ali khalaf hussain**

[1]College of Education for Pure Sciences, Wasit University, Iraq

*Corresponding Author: Marwa Munther Hassan

DOI: https://doi.org/10.31185/wjcm.51

Received: June 2022; Accepted: August 2022; Available online: September 2022

ABSTRACT: In this paper, we introduce the concept of SPG*-closed mapping and continuous mapping among which SPG-closed mappings, SPG*-closed mappings and SPG**-closed mappings and the relationship between them, and also SPG-continuous mappings, SPG*-continuous mappings and SPG**-continuous map-pings and the relationship between them. Among the result we obtain are the concepts of SPG**-closed mappings and closed mappings are independent also PG**-continuous mappings and continuous mapping are independent.

Keywords: spgclosed set, SPG*closed map, SPG**closed map, SPG continuous map, SPG*continuous map and SPG**continuous map

CC BY

1. INTRODUCTION

The notions of **semi-opensets** were introduced and studied by Levine [1] in 1963.In 1970, Levine [2] initiated and studied **generalized-closed sets**. and **generalized-open sets** as **generalization of closed sets** and **open sets**. Dunham [3] came up with the concept of **generalized closure** using Levine's **generalized closed sets** and interpreted its properties. The investigation of **generalized closed sets** has led to several interesting concepts in topology. Recently, the topologists studied various **generalized closed sets** in **topological spaces**. In 1982, Mashhour, Abd El-Monsef and El-Deeb [4] defined **pre-open sets** and **pre-continuous functions**. The class of **pre generalized-closed sets** that were used to obtain properties of **pre-T**$_{1/2}$**spaces** were introduced by Maki, Umehara and Noiri [5] in 1996.In 1986 D. Andrijevic [6] introduced and studied **semi-pre opensets** in topology. Later, many authors have been studied these **semi-pre open sets** and **semi-pre closed** and **generalized-continuous functions**. In 1995 Dontchev [7] defined **generalized semi-pre open sets** (in briefly gsp-open Sets).In [8], Maki introduced the concepts of **pg-closed** sets and **gp-closed** sets in an analogous manner. These notions are generalizations of **pre-closed** sets which were further studied by Dontchev and Maki [9], leading to a new decomposition of **pre-continuity**. In this paper, our aim is to introduce the concept of **SPG*-closed** map and **SPG*-continuous** map in **topological spaces** and studied the relationship between them and investigate their basic properties.

2. BASIC CONCEPTS

Definition 2.1

Let (X,T) be a topological space and let B⊆X then the set B is called:

1. $semi\ \ pre-closed\ \ set$(briefly, $sp-closed$) if $\overline{((\overline{B^o}))^{o}}\ \ B$, The complements of $semi\ \ pre-closed\ \ set$ is a $semi\ pre-openset$(briefly, $sp-open$) and represent that $B\ \ ((\overline{B})^{o})$ [10].

2. $generalized-closed\ set$ (briefly, $g-closed$) if $cl(B)\subseteq V$, whenever $B\subseteq V$ and V is an $open\ set$. The complement of a $generalized-closedset$ is called a $generalized-openset$ (briefly, $g-open$) and represent that $cl(B)\ \ \subseteq V$, whenever

$B \subseteq V$ and V is a $closedset$ [11].

3. *pre generalized−closedset* (briefly, *pg−closed*) if $pcl(B) \subseteq V$, whenever V is a *pre−open* subset of X and $B \subseteq V$. The complements of $pg - closedsets$ is *pre generalized − open set*(briefly, *pg − open*) [12].

4. *semi pre generalized−closedset* (briefly, *spg−closed*) if $spcl(B) \subseteq V$ whenever $B \subseteq V$ and V is a semi *pre−openset* in X.The complements of $spg - closedsets$ are called *semi pre generalized − opensets* (briefly, *spg − open*).

Remark 2.1

1. Every spg-closed set is a sp-closed\;set and the converse is true
2. Every $closedset$ be a $pg - closedset$, but the converse does not necessary to be true.

Definition 2.2

Let (X, T) be a *topological space*, we say that X is a *submaximal space* if every $sp - openset$ in X is an *open set*.

Proposition 2.1

Let (X, T) be a *submaximal space*, then every $sp - closed$ set in X is a $closedset$.

Proof.

Let $B \subseteq X$ is $sp - closedset$

Then B^c is $sp - openset$

And since X be a *submaximal space*

So B^c is $openset$ [2.3]

Hence B is a $closedset$.

Through remark 2.1(1), we clarify the relationship between $spg - closedset$ and $sp - closedset$ and from the above proposition 2.1,.we can get the followingscorollary:

Corollary 2.1

Let (X, T) be a *submaximal space*, then every $spg - closedset$ in X be a $closedset$.

Definition 2.3

Let X and Y be a *topological spaces*, the $mapf : X \to Y$ is called *closed map*(*open*) if each *closed* (*open*) $setO \subseteq X$ then $f(O)$ be a *closed* (*open*) *set* in Y. [13]

Definition 2.4

Let X and Y be a *topological spaces*, we say that $f : X \to Y$ is $SPG - closed\ map$ ($SPG - open$) if each *closed* (*open*)*set* $O \subseteq X$ then $f(O)$ is $spg - closed$ ($spg - open$)*set* in Y.

Remark 2.2

Every *closed* (*open*)*map* is a $PG - closed$ ($SPG - open$)*map*, "but the convers does not necessary to be true", as we will explain through the following examples:

Example 2.1

Let = $\{1, 2, 3, 4, 5\}$, $T = \{X, \varnothing, (3, 4)\}$ be a *topological* defined on X.

And Let = $\{a, b, c, d, e\}$, $\delta = \{Y, \varnothing, \{a, c, d\}\}$ be a *topological* defined on Y we defined a $mapf : (X, T) \to (Y, \delta)$ as follows:

$f(1) = a$, $f(2) = b$, $f(3) = c$, $f(4) = d$, $f(5) = e$

It is clear that f is $SPG - closed\ map$ ($SPG - open$), but f is not closed *map*

Since $\{1, 2, 5\}$ is a $closedset$ in X but $f(\{1, 2, 5\}) = \{a, b, e\}$ is not *closed set* in Y.

And $\{3, 4\}$ is an $openset$ in X but $f(\{3, 4\}) = \{c, d\}$ is not *open* in Y.

Remark 2.3

If Y be a *submaximal space* then f be a *closed map*, as we will clarify.through the following theorem:

Theorem 2.1

Let each of X and Y be a *topological space*, let $f : X \to Ya$ be a $SPG - closedmap$ if Y is *submaximal space* then $f : X \to Ya$ is a *closed map*.

Proof.

Let f be a $SPG - closed\ map$

Let $B \subseteq X$ be a *closed set*

Since $f : X \to Y$abe a $SPG - closed\ map$

so $f(B)$ is $spg - closed\ set$ in Y

And since Y is *submaximal space*

So $f(B)$ is $closedset$ in Y [by corollary 2.1]

Hence $f : X \to Y$ is a *closed map*.

Definition 2.5

Let each of X and Y be a *topological spaces*, we say that $f : X \to Ya$is $SPG* -closed\ map$ ($SPG* -open$) if each $spg - closed\ set$ ($spg - open$)$B \subseteq X$ then $f(B)$ is *closed* (*open*)*set* in Y.

Example 2.2

Let $X = \{1, 2, 3\}$ and let each of $T = \{X, \emptyset, \{1\} (2, 3\}\}$
And $\delta = \{Y, \emptyset, \{1\}, (2\}, (3\}, (1, 2\}, \{1, 3\}\{2, 3\}\}$ be a *topologies* defined on X.
we defined a $map f : (X, T) \to (X,\ \delta)$ as follows:
$f(1) = f(2) = 1$, $f(3) = 3$
So f is $SPG*-closed\ (SPG*-open)$ map.

Definition 2.6

Let each of X and Y be a *topological spaces*, then $f : X \to Y$ is $SPG**-closed\ map\ (SPG**--open)$ if each $spg-closed\ \ set(spg-open) B \subseteq X$ then $f(B)$ is $spg-closed\ set(spg-open)$ in Y.

Example 2.3

Let $X = \{1, 2, 3\}$, $T = \{X, \emptyset, (1\}, (2, 3\}\}$ be a *topological* defined on X.
And Let $Y = \{a, b, c\}$, $\delta = \{Y, \emptyset, (a\}, \{b, c\}\}$ be a *topological* defined on Y
we defined a map $f : (X, T) \to (Y,\ \delta)$ as follows:
$f(1) = a$, $f(2) = b$, $f(3) = c$
So f is $SPG**-closed\ \ (SPG**-open)\, map$.

Definition 2.7

The map $f : X \to Y$ is called bijective map if it is injective and surjective. [14]
It is known that if $f : X \to Y$ is a bijective map then f is an *open* map if and only if f is a *closed* map .
We can *generalize* this to $SPG-closed$ map and $SPG*-closed$ and $SPG**-closed\ map$, as we will explain through the following theorems:

Theorem 2.2

Let $f : X \to Y$ be a bijective map then:
1. f Be a SPG-closed map if and only if f is SPG-open map
2. f Be a SPG *-closed map if and only if f is SPG *-open map
3. f Be a SPG **-closed map if and only if f is SPG ***-open map

Proposition 2.2

Every $SPG*-closed$ map be a $closed\ map$.

Proof.

Suppose that $f : (X, T) \to (Y,\ \delta)$ is $SPG*-closed$ map
Let $B \subseteq X$ is a $closedset$
So B is $spg-closedset$ [by remark 2.1 (2)]
Since f is $SPG*-closed\ map$, so $f(B)$ is $closedset$ in Y
Hence f is a $closed\ map$

Remark 2.4

The convers of the proposition 2.2, does not necessarily to be true , as we will explain through the following examples:

Example 2.4

Let $X = \{1, 2, 3\}$, and let $T = \{X, \emptyset, (2\}, (2, 3\}\}$ be a *topology* defined on X.
we defined a $map f : (X, T) \to (X, T)$ as follows:
$f(1) = f(2) = 1$, $f(3) = 3$
So f is a $closed\ map$ but f is not a $SPG*-closed\ map$
Since $\{3\}$ is a $spg-closedset$ in (X, T) and $f(3) = \{3\}$ is not $closedset$ in (X, T).

Corollary 2.2

Every $SPG*-closed\ map$ be a $SPG-closed\ map$.

Remark 2.5

The convers of the corollary 2.2, does not necessarily to be true , as we will explain through the following examples

Example 2.5

Let $X = \{1, 2, 3, 4, 5\}$, $T = \{X, \emptyset, (1\}, (3, 4\}, (3, 4\}\}$ be a *topological* defined on X.
And Let $Y = \{a, b, c, d, e\}$, $\delta = \{Y, \emptyset, (a\}, \{c, d\}\{a, c, d\}\}$ be a *topological* defined on Y
we defined a $map f : (X, T) \to (Y,\ \delta)$ as follows:
$f(1) = a$, $f(2) = b$, $f(3) = c$, $f(4) = d$, $f(5) = e$
So f is a $SPG-closed\ map$, but f is not $SPG*-closed\ map$ Since $\{3, 4\}$ is a $spg-closedset$ in X and $f(\{3, 4\}) = \{c, d\}$ is not $closedset$ in Y.

Proposition 2.3

Every $SPG**-closed\ map$ be a $SPG-closed\ map$.

Proof.

Suppose that $f : (X, T) \to (Y,\ \delta)$ is $SPG**-closed\ map$
Let B be a $closedset$ in X

So B is $spg - closed\ \ set$ in X [by remark 2.1 (2)]
Since f is $SPG ** - closed\ map$
So B is a $spg - closedset$ in Y
Hence f is a $SPG - closed$ map.

Remark 2.6

The convers of Proposition 2.3, does not necessarily to be true, as we will explain through the following examples:

Example 2.6

Let $f : (R, I) \to (R,\ T_u)$ be a map defined as follows:
$\forall x \in X\ ,\ \ f(x) = xa$
So f is a $SPG - closed\ map$, but f is not $SPG ** - closed\ map$
Since $(a\}$ is a $spg - closedset$ in (R, I) and $f(\{a\}) = \{a\}$ is $spg - closed\ set$ in (R, T_u) .

Proposition 2.4

Every SPG*-closed mapbe a SPG**- closed map.

Proof.

Assume that each of X and Y are $a\ topological\ space$
Since f is $SPG * -closed$ map
So $f(B)$ is a $closedset$ in Y
Hence $f(B)$ a $spg - closedset$ in Y [by remark 2.1 (2)]
So f is $SPG ** - closedmap$

Remark 2.7

The convers of Proposition 2.27, does not necessarily to be true, for example:

Example 2.7

Let $X = \{1, 2, 3\}$, $T = \{X, \varnothing, (1\}, (2, 3\}\}$ be a $topology$ defined on X.
And Let $Y = \{a, b, c\}$, $\delta = \{Y, \varnothing, (a\}, \{b, c\}\}$ be a $topology$ defined on Y
We defined a $mapf : (X, T) \to (Y,\ \delta)$ as follows:
$f(1) = a\ \ , f(2) = b\ \ ,\ f(3) = c$
So f is a $SPG ** - closed\ map$
And Since $\{1, 3\}$ is a $spg - closedset$ in X and $f(\{1, 3\}) = \{a, c\}$ is not $closed\ set$ in Y.
So f is not $SPG * -closed\ map$.

Remark 2.8

As for the relationship between the concept of the SPG**-closed mapand the concept of the closed map, one does not the other as we will show through the following examples:

Example 2.8

Let $f : (R,\ T_u) \to (R, I)$ be a map defined as follows:
$\forall x \in R\ ,\ \ f(x) = xa$
So f is a $SPG ** - closed\ map$, but f is not $closed\ map$
Since $B = [0, b]$ is a $closedset$ in (R, T_u) and $f(B) = [0, b]$ is not a $closed\ set$ in (R, I) .
Now can write the following diagram to show the relationship between $SPG - closed\ map$ and $SPG * -closed\ map$ and $SPG ** - closed\ \ map$:
SPG**-$closed\ map$

Diagram (1). explain the relationship between $SPG * -closed\ mapping$**.**

Now we study the SPG-continuous map and SPG*-continuous map and SPG**-continuous map.

Definition 2.8

The $mapf : X \to Y$ is called $continuous\ map$ if every $closed\ \ set(open)D$ in Y, then $f^{-1}(D)$ be a $closed\ set(open)$ in X. [13]

Definition 2.9

Let each of X and Y be $a\ topological\ spaces$, then $f : X \to Y$ is called $SPG\ continuous\ map$ if every $closed\ set(open)D$ in Y then $f^{-1}(D)$ be a $spg - closed\ set$ in X .

Remark 2.9

Every $continuousmap$ be a $SPG -\ continuous\ map$, but the convers does not necessarily to be true for examples:

Example 2.9

Let $X = \{1, 2, 3\}$ and let each of $T = \{X, \varnothing\}$ and $\delta = \{X, \varnothing, (1\}\}$ are a $topologies$ defined on X.
We defined a $mapf : (X, T) \to (X,\ \delta)$ as follows:
$\forall x \in X,\ \ f(x) = xa$
So f is a $SPG -\ continuous\ map$ since (f^{-1}) for every $closedset$ in $(X,\ \delta)$ be $spg - closedset$ in $(X,\ \delta)$, but f is not $continuous\ map$ since $\{2, 3\}$ is a $closed\ set$ in $(X,\ \delta)$ but $f^{-1}((2, 3\}) = \{2, 3\}$ is not $closedset$ in (X, T).

Theorem 2.3

Let $f : X \rightarrow Y$ be a $SPG-$ $continuous\ map$, if X is $submaximal\ space$ then f is $continuous\ map$.

Proof .

Assume that B a $closedset$ in Y

Since f is a $SPG-$ $continuous\ maps$

Then $f^{-1}(B)$ is a $spg - closedset$ in X

And since X is a $submaximal\ space$

So B is a $closedset$ in X

Thus f is a $continuous\ map$.

Definition 2.10

Let each of X and Y be a $topological\ spaces$, then $f : X \rightarrow Y$ is called

$SPG* - continuous\ map$ if every $spg - closed\ set(spg - open)$ D in Y then $f^{-1}(D)$ be a $closed\ set$ $(open)$ in X .

Example 2.10

Let $X = \{1, 2, 3\}$ and let $T = \{X, \emptyset, (1\}, (2\}, (3\}, (1, 2\}, (1, 3\}, \{2, 3\}\}$ and

$\delta = X, \emptyset, (1\}, \{2, 3\}\}$ are a $topologies$ defined on X.

We defined a map $f : (X, T) \rightarrow (X,\ \delta)$ as follows:

$f(1) = f(2) = 1$, $f(3) = 3$

So f is a $SPG* - continuous\ map$.

Remark 2.10

Every $SPG* - continuous\ map$ be a $continuousmap$, but the convers does not necessarily to be true for examples:

Example 2.11

Let $f : (R, T_u) \rightarrow (R,\ \delta)$ be a map defined as follows:

$\forall x \in R$, $f(x) = xa$

So f is a $continuous\ map$, but f is not a $SPG* - continuous\ map$

Since $(b\}$ is $spg - closedset$ in δ and $f^{-1}((b\}) = \{b\}$ is not a $closedset$ in T_u .

Definition 2.11

Let each of X and Y be a $topological\ spaces$, then $f : X \rightarrow Y$ is called $SPG** -$ $continuous\ map$ if every $spg - closed\ set$ $(spg - open)$ D in Y then $f^{-1}(D)$ be a $spg - closed\ set(spg - open)$ in X .

Example 2.12

Let $f : (X, I) \rightarrow (Y,\ \delta)$ be $a\ map$ defined as follows:

$\forall a \in X$, $f(a) = aa$

So f is $a\ SPG** -$ $continuous\ map$.

Remark 2.11

Every $SPG** -$ $continuous\ map$ be a $SPG - continuous\ map$, but the convers does not necessarily to be true for example:

Example 2.13

Let $f : (R, T_u) \rightarrow (R,\ I)$ be a map defined as follows:

$\forall x \in R$, $f(x) = xa$

So f is a $SPG - continuous\ map$, but f is not a $SPG** -$ $continuous\ map$

Since $(a\}$ is $spg - closedset$ in (R, I) and $f^{-1}(a) = \{a\}$ is not a $spg - closedset$ in (R, T_u)

Remark 2.12

Every $SPG* - continuous\ map$ be a $SPG** - continuous\ map$, but the convers does not necessarily to be true for example:

Example 2.14

Let $aX = \{1, 2, 3\}$ and let each of $T = \{X, \emptyset\}$ and $\delta = \{X, \emptyset, (1\}, \{2, 3\}\}$ are a $topologies$ defined on X.

We defined a $mapf : (X, T) \rightarrow (X,\ \delta)$ as follows:

$\forall x \in X$, $f(x) = xa$

So f is a $SPG** -$ $continuous\ map$ and f is not a $SPG* - continuous\ map$

Since $(1\}$ is $spg - closedset$ in $(X,\ \delta)$ and $f^{-1}((1\}) = \{1\}$ is not a $closed\ set$ in (X, T) .

Corollary 2.3

Through the forgoing, we can know that the concept of SPG**- continuous map is independent of the concept of continuous map.

Now can write the following diagram (2), to show the relationship between a SPG- continuous map and SPG*-continuous map and SPG**- continuous map

SPG∗∗-Continuous map

Diagram (2). explains relationship between $SPG - closed\ map$ and $SPG* -closed\ map$ and $SPG** - closed\ map$.

3. CONCLUSION

The concept of $SPG**-closed\ map$ is independent of the $closed\ map$ and also the $SPG**-continuous\ map$ a concept independent of the $continuous\ map$.And also conclusion every SPG*-closed map (continuous map) is a SPG**-closed map (continuous map) , and SPG**-closed map (continuous map) is a SPG-closed map (continuous map) , and the converse does not necessary to be true.

FUNDING

None

ACKNOWLEDGEMENT

None

CONFLICTS OF INTEREST

The author declares no conflict of interest.

REFERENCES

[1] A. Alkhazragy, A. K. H. A. Hachami, and F. Mayah, "Notes on strongly Semi - closed graph," *Herald of the Bayman Moscow StateTechnical University, Series Natural Sciences*, pp. 17–27, 2022.
[2] M. Marwah, A. Hassan, and K. Hussain, "On Semi pre-generalized-closed sets," *Wasit journal for pure science*, vol. 1, no. 2, pp. 2022–2022.
[3] Dunham, "A new closure operator for non-T1 topologies," *Kyungpook Mathematical Journal*, vol. 22, no. 1, pp. 55–60.
[4] A. S. Mashhour, A. El-Monsef, M. , and El-Deeb, "On pre continuous and weak pre continuous mappings," *SN 1982*, vol. 53, pp. 47–53.
[5] H. Maki, Umehara, and T. Noiri, "Every topological space in pre-T1/2," *Memoirs of the Faculty of Science Kochi University Series A Mathematics*, vol. 17, pp. 33–42, 1996.
[6] R. Nithyakala and A. Pushpalatha, "Semi(c (s))-Generalized Closed Sets in Topological Spaces," *Journal of Informatics and Mathematical Sciences*, vol. 9, no. 3, pp. 99–112, 1971.
[7] H. Maki, J. Umehara, and T. Noiri, "Every topological space is pre-T12," *Mem. Fac. Sci. Kochi Univ. Ser. A, Math*, vol. 17, pp. 33–42, 1996.
[8] J. Dontchev and H. Maki, "On the behavior of gp-closed sets and their generalizations," *Mem. Fac. Sci. Kochi Univ. Ser A, Math*, vol. 19, pp. 57–72, 1998.
[9] A. El-Monsef, M. E. El-Deeb, S. N. Mahmoud, and R. A, "b-Open Sets and b-Continuous Mappings," *Bull. Fac. Sci. Assiut. Univ*, vol. 12, pp. 77–90, 1983.
[10] S. Maragathavalli and Sathiyavathi, "Strongly Generalized closed sets in."
[11] "Ideal Topological Spaces," *International Journal of Physics and Mathematical Research*, vol. 3, no. 2, pp. 11–013, 2014.
[12] J. Cao and M. G. I. Reilly, "On Preclosed Set And Their Generalizations" ,To Appear In Houston," *J.Math*.
[13] L. K. A. R. Al-Jalabi, ed., *On Closed Applications), Master's Thesis*. 1998.
[14] J. N. Sharma and J. P. Chanhan, *Topology General and Algebraic*. 2016.