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# Some approaches to solving fuzzy linear fractional programming

**Waleed Mohammed Elaibi [1], Hadi Nasseri [2]**

[1]Department of Statistics, College of Administration and Economics, Mustansiriyah University, Baghdad, waleed_m@uomustansiriyah.edu.iq, Iraq

[2]Department of Applied Mathematics, University of Mazandaran, Babolsar, hadi57@gmail.com, Iran

*Corresponding Author: Waleed Mohammed Elaibi

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**ABSTRACT:** An important planning tool is linear fuzzy fractional programming, it is used in various fields such as business, engineering, and others. We are trying to get one of the direct and effective methods that contain some arithmetic operations to obtain the optimal real values through which a multi-objective fuzzy linear programming problem (MOFLFPP) is transformed into a linear programming problem (LPP) through the use of α-cut and Max-Min technique.
The field of application is Iraqi Light Industries Company and chose the best products that must be protected which achieved a possible greatest profit ratio to less cost, Where the paper will include two sections, the first is concerned with describing the data and building the mathematical model for the problem (MOFLFPP) related to the research problem. The second section deals with trying to solve the model as well as finding the optimal solution, which represents determining the best and optimal production mix that achieves maximum profits at the lowest costs in light of the restrictions imposed on the production process, which may limit the company's ability to provide products in the required quantity and the right time.
The proposed methodology proved effective in solving multi-objective linear fractional programming problems with fuzzy coefficients (MOLFPP). While the previous approach produced results between (0.19904, 0.3406), our technique improved them to (0.2087, 0.3431), demonstrating higher reliability and efficiency with an ε-optimal unique solution.
**Keywords:** Fuzzy sets, Linear programming, Linear fractional programming, Fuzzy coefficients.



## 1. INTRODUCTION:

The need for computerized scientific research arose as a result of the quick advancements in science at the level of educational, manufacturing, and service institutions, particularly in operations research, where it is necessary to use computerized applications to study program changes that result from significant changes in plans and programs. The majority of the challenges faced by decision-makers arise when there are multiple goals to accomplish a given problem, which is one of the most significant objectives of operations research and hence ranks first in decision-making, and when we encounter a situation where two goals are incompatible, one of them is relative to the other goal, requiring that one be maximized and the other be minimized. Herein lies the significance of linear fractional programming and nonlinear fractional programming, which are regarded as two of the most important topics for solving such cases and have a wide range of applications in this field. They aim to maximize various factors, such as production relative to the workforce, production relative to costs, production relative to manufacturing process waste, and others.

The figure (1) defines the triangular fuzzy number $\widetilde{A} = (a_1, a_2, a_3)$ along with its membership function. Using the $\alpha - cut.$, the fuzzy number is represented as a closed interval of real numbers, enabling arithmetic operations. These operations include addition, scalar multiplication, multiplication, and division based on the lower and upper bounds of the intervals.
Outline the field of application:

The Light Industries Company was selected as the field of application. The company is located in Al-Zafaraniya, Baghdad, and is one of the mixed-sector companies. It was established in 1959 with a nominal capital of half a million dinars. The company has evolved over the years, and its current capital is 16.8 billion Iraqi dinars.

The company's area is approximately 283,000 square meters and includes three factories:

1- Refrigerator factory, which produces various types of refrigerators in different sizes with an area of about 30,000 square meters.
2- Oil heaters and gas cookers factory with an area of about 25,000 square meters.
3- Freezers factory, which produces various types of freezers with an area of about 20,000 square meters.

## 2. Literature Review

several academics have studied fuzzy linear fractional programming. Alharbi, M. G., & Khalifa, H. A. (2021) provided a method for converting the maximizing (minimizing) problem with an interval objective function into a multi-objective problem based on order relations established by the decision makers' choice between interval profits (costs). Abd El-Wahed Khalifa, H., Kumar, P., & Alodhaibi, S. S. (2022) Analyze a multi-objective linear fractional programming (FMOLFP) Problem using fuzzy pseudorandom decision variables and fuzzy random coefficients. First, a single objective fuzzy linear programming (FLP) model is created from the FMOLFP model. Second, we demonstrate how a fuzzy random optimum FLP solution resolves into a class of random optimal LP solutions using the relative pseudorandom LP model. A few theorems demonstrate that a fuzzy random optimal solution to a fuzzy pseudorandom LP Problem is paired with a number of relative fuzzy random optimal solutions to pseudorandom LP problems. Ammar, E. S., & Muamer, M. (2016) offer a method for handling multi-objective linear fractional programming problems where the objective functions have a fuzzy coefficient (MOFRLFP). It is assumed that every parameter of the goal functions has a fuzzy triangle number. To address the aforementioned problem, the first algorithm uses the (a-cut) technique, and the second algorithm uses the ranking function. Amer, A. H. (2018) is creating a novel interactive approach based on intuitionistic fuzzy set theory to solve nonlinear fractional programming problems. Additionally, MONLFPP can be converted to a single objective non-linear programming problem (NLPP) using the fuzzy mathematical programming approach, which can then be easily solved using any appropriate NLPP algorithm. Ammar, E. S., & Muamer, M. (2016) Since all of the variables and coefficients of the objective function and constraint are fuzzy numbers, the (FRLFP) problem can be simplified to the multi-objective fuzzy linear fractional programming (MOFLFP) problems. furthermore, obtaining a best-case fuzzy rough answer utilizing the decomposition algorithm. As Suggested by Borza, M., & Rambely, A. S. (2021) method, It converts the MOLFPP into a linear programming problem. and it has been demonstrated that the LPP's ideal solution is an active one for the (MOLFPP). Chakraborty, M., & Gupta, S. (2002) presented a similar form of the issue that has been formulated in the suggested methodology in terms of multi-objective linear programming. The fuzzy set-theoretic method has been used to investigate a procedure. Numerous numerical examples have also been solved using the suggested solution method. Chakraborty, M., & Gupta, S. (2002) provided a method for solving Problems involving multi-objective linear fractional programming. The Problem has been formulated in an equivalent manner for multi-objective linear programming. utilizing a theoretically fuzzy method. In order to resolve numerical cases, the proposed solution method was also applied. Chinnadurai, V., & Muthukumar, S. (2016) For a linear fractional programming issue with ambiguous coefficients and decision variables, it was advised to use the usable optimum value (α, r). also creating a mechanism for calculating them. We apply an (α-cut) on the objective function and r cut on the constraints to arrive at acceptable (α, r) optimum values. Then, in order to determine the upper and lower bounds of the fully fuzzy (LFP) problem, we create an equivalent bi-objective linear fractional programming problem. We numerically generate the membership functions of the optimal values using the upper and lower constraints that were established. Das, S. K., Edalatpanah, S. A., & Mandal, T. (2021) In order to formulate a linear fractional programming (LFP) problem, we assume that the costs of the constraints and objective functions are both triangular fuzzy numbers. The centroid ranking function is used to convert the (LFP) problem into an analogous crisp line fractional programming (CLFP) problem. This suggested approach has a straightforward structure and is based on crisp (LFP). A real-life problem has been demonstrated to demonstrate the effectiveness of our suggested approach. Decision-makers will understand the utility of the (CLFP) problem thanks to the presentation of the practical problem. Das, S. K., Edalatpanah, S. A., & Mandal, T. (2020) recommended a fresh and efficient approach to solving (FFLFP) problems encountered every day. The suggested method has a straightforward structure and is based on linear split programming. Several numerical and real-world Problems have been demonstrated to demonstrate the efficacy of our suggested approach. Das, S. K., & Edalatpanah, S. A. (2020) developed a strategy using ranking and decomposition techniques to address the Problem of fractional linear programming in a fuzzy environment. This investigation has led to the conclusion that LU linear decomposition equations cannot be used to solve fuzzy linear fractional programming problems with equality requirements. Either adding slack variables or applying equality constraints can fix this. Das, S. K., Edalatpanah, S. A., & Mandal, T. (2018) suggestions for a simple ranking algorithm between two fuzzy triangular integers. Create an identical tri-objective linear fractional programming problem to identify the fuzzy linear fractional programming issue's upper, middle, and lower limits (FLFPP). In addition, In addition, using the upper, middle, and lower bounds, we numerically construct the ideal values. Das, S. K., & Mandal, T. (2017) suggested a new efficient method for the FLFP problem. These suggested approaches are based on crisp linear fractional programming and a new transformation technique is also used. A calculation procedure was presented in order to arrive at an optimal solution. Das, A method for solving the Fully Fuzzy Linear Fractional Programming (FFLFP) problem, in which all of the variables and parameters were fuzzy triangular numbers, was proposed by S. K. and Mandal, T. (2017). Furthermore, the FFLFP has been transformed into a multi-objective linear fractional programming problem (MOLFPP) counterpart. Then, using a mathematical programming technique, MOLFP was transformed into an analogous multiobjective linear programming problem.. A fuzzy environment for multiobjective linear fractional programming problems (MOLFPP) was proposed by Das, S. K., & Mandal, T. (2017). An uncertain level of aspiration is added to each goal in order to transform a multiobjective linear fractional programming into a multiobjective linear programming issue using the first-order Taylor series approach. The solution was then obtained using the additive weighted technique. It was noted that optimisation was accomplished for various membership function weight values for various objective functions. A novel solution to the multi-objective linear fractional programming problem (MOLFP) was proposed by Güzel, N. (2013, September). The proposed approach is based on a theorem that addresses nonlinear fractional programming with a single objective function. They proposed the innovative idea that if x is the best solution to the problem, then x is an efficient solution of (MOLFPP). The linear programming problem (LPP) is hence the straightforward simplification of (MOLFPP).

## 3. Some Concepts and Notions

### 3.1 Intervals and fuzzy numbers

***Def. (3.1.1)*** Let $\widetilde{A}$ which was a normalized fuzzy set. $\widetilde{A}$ is the triangular fuzzy number and defined as:

$$\mu_{\widetilde{A}}(x) = \mu_{\widetilde{A}}(x, a_1, a_2, a_3) = \begin{cases} (x - a_1)/(a_2 - a_1), & x \in [a_1, a_2) \\ (a_3 - x)/(a_3 - a_2), & x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

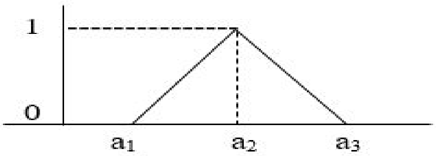


Fig. (1) triangular fuzzy number

Due to the fact that the (α-cut) of any fuzzy number is a closed interval of real numbers, (α-cut) fully and exclusively represents a fuzzy number. Because of this, we are able to describe arithmetic operations on the fuzzy number in terms of α reductions. Given two fuzzy sets $\widetilde{A} = \left[A_\alpha^{LB}, A_\alpha^{UB}\right]$ and $\widetilde{B} = \left[B_\alpha^{LB}, B_\alpha^{UB}\right]$. The following is a definition of the arithmetic operations:

(I) Addition: $\left(\widetilde{A} + \widetilde{B}\right) = \left[A_\alpha^{LB} + B_\alpha^{LB}, A_\alpha^{UB} + B_\alpha^{UB}\right]$.

(II) Scalar multiplication: $\left(k\widetilde{A}\right)_\alpha = \left[kA_\alpha^{LB}, kA_\alpha^{UB}\right]$, if $k\rangle 0$ and $\left(k\widetilde{A}\right)_\alpha = \left[kA_\alpha^{UB}, kA_\alpha^{LB}\right]$,if $k\langle 0$.

(III) Multiplication $\left(\widetilde{A}.\widetilde{B}\right) = \begin{bmatrix} Min\left(A_\alpha^{LB}B_\alpha^{LB}, A_\alpha^{LB}B_\alpha^{UB}, A_\alpha^{UB}B_\alpha^{LB}, A_\alpha^{UB}B_\alpha^{UB}\right), \\ Max\left(A_\alpha^{LB}B_\alpha^{LB}, A_\alpha^{LB}B_\alpha^{UB}, A_\alpha^{UB}B_\alpha^{LB}, A_\alpha^{UB}B_\alpha^{UB}\right) \end{bmatrix}$

(IV) Division: $$\left(\frac{\widetilde{A}}{\widetilde{B}}\right)_\alpha = \begin{bmatrix} Min\left(\frac{A_\alpha^{LB}}{B_\alpha^{LB}}, \frac{A_\alpha^{LB}}{B_\alpha^{UB}}, \frac{A_\alpha^{UB}}{B_\alpha^{LB}}, \frac{A_\alpha^{UB}}{B_\alpha^{UB}}\right), \\ Max\left(\frac{A_\alpha^{LB}}{B_\alpha^{LB}}, \frac{A_\alpha^{LB}}{B_\alpha^{UB}}, \frac{A_\alpha^{UB}}{B_\alpha^{LB}}, \frac{A_\alpha^{UB}}{B_\alpha^{UB}}\right) \end{bmatrix}.$$

***Def. (3.1.2)*** *According* to (*Ref.* [4, 8]) Suppose that we are a fuzzy set in X and $\alpha$ belong to [0, 1]. $\widetilde{A}$ is the crisp set $\widetilde{A}_\alpha$ by $\alpha - cut$.

$$[\widetilde{A}_\alpha] = \{x \in X : \mu_{\widetilde{A}}(x) \geq \alpha\}$$

The membership function $\mu_{\widetilde{A}}(x) = \mu_{\widetilde{A}}(x, a_1, a_2, a_3)$ and $\widetilde{A}$ be a triangular fuzzy number, then

$$[\widetilde{A}_\alpha] = [a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)] \quad (1)$$

***Def. (3.1.3)*** (***Ranking method of fuzzy numbers***) If we assume $\left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\right)$ they are fuzzy numbers with $\alpha - cut$. $\left[\tilde{A}_1\right]_\alpha = \left[a_{1\alpha}^-, a_{1\alpha}^+\right], \left[\tilde{A}_2\right]_\alpha = \left[a_{2\alpha}^-, a_{2\alpha}^+\right], \left[\tilde{A}_3\right]_\alpha = \left[a_{3\alpha}^-, a_{3\alpha}^+\right]$. based on $\alpha - cut$ can be used probability and necessity theories to rank fuzzy numbers as follows:

**Method (I).** Say $\tilde{A}_2$ greater than or equal to $\tilde{A}_1$ and denoted by $\tilde{A}_1 \leq \tilde{A}_2$ if and only if $a_{1\alpha}^- \leq a_{2\alpha}^-$, and $a_{1\alpha}^+ \leq a_{2\alpha}^+$ for $\alpha \in (0,1]$. in addition to, for $k_1, k_2 \geq 0$ we say $k_1\tilde{A}_1 + k_2\tilde{A}_2 \leq \tilde{A}_3$ if and only if $k_1 a_{1\alpha}^- + k_2 a_{2\alpha}^- \leq a_{3\alpha}^-$, and $k_1 a_{1\alpha}^+ + k_2 a_{2\alpha}^+ \leq a_{3\alpha}^+$.

**Method (II).** Say $\tilde{A}_2$ greater than or equal to $\tilde{A}_1$ and denoted by $\tilde{A}_1 \leq \tilde{A}_2$ if and only if $a_{1\alpha}^+ \leq a_{2\alpha}^+$ for $\alpha \in (0.5,1]$. in addition to, for $k_1, k_2 \geq 0$ we say $k_1\tilde{A}_1 + k_2\tilde{A}_2 \leq \tilde{A}_3$ if and only if $k_1 a_{1\alpha}^+ + k_2 a_{2\alpha}^+ \leq a_{3\alpha}^+$

### 3.2 Linear Fractional programming (LFP)

*According* to (*Ref.* [7, 8, 9]) This section will cover the general form of the (LFP) that we will enforce, which is as follows:

$$Maximize\left\{(Z)\,\frac{C^{num}X + \theta_1}{D^{den}X + \theta_2}\right\} \qquad (2)$$

s.t $AX \leq b,\ D^{den}X + \theta_2 > 0,\ X \geq 0,\ X, C^{num}, D^{den} \in R^n,\ A \in R^{mxn},\ \theta_1, \theta_2 \in R$

Following Charnes and Cooper's approach Equation (2) is transformed into the following linear Problem utilizing variable transformations, in accordance with (*Ref.* [7, 8]).

$$t = \frac{1}{D^{den}X + \theta_2}, Y = tX.$$

$$Maximize \quad \left\{C^{num}Y + \theta_1(t)\right\} \qquad (3)$$

$$AY - Nb(t) \geq 0,\ D^{den}Y + \theta_2(t) = 1,\ Y, t \geq 0$$

***Theorem (2.4.)*** The optimal solution for equation (3) by imposition $(Y^{(*)}, t^{(*)})$, is based on that the $X^{(*)} = \left(\frac{Y^{(*)}}{t^{(*)}}\right)$ an optimal solution to equation (2).

### 3.3 Multi-Objective Programming Problem (MOPP)

The general form of the multi-objective linear programming problem is as follows:

$$Maximize\{F_1(X), F_2(x), F_3(X), \ldots\ldots\ldots, F_n(X)\} \qquad (4)$$

s.t $X \in S$

***Definition (2.5.)*** (*Ref.* [7]) $X^{(*)} \in S$ is a solution and should be called efficient if and only if $X \in S$ is not exists such that $F_j(X^{(*)}) \leq F_j(X),\ j = 1,2,3,\ldots, k$ and $l \in \{1,2,\ldots, k\}$ is exists such that $F_j(X^{(*)}) < F_j(X)$

The (max-min) approach is a traditional method that is can be used to secularize the (MOPP) as follows:

$Maximize(\beta)$ s.t $X \in S$, $\beta \leq F_i(X)$ for i=1,2,,k (5)

***Definition (2.6.)*** (*Ref.* [8]) Let's think a little bit about the problem of a single-objective $\underset{X \in S}{Maximize}\ g(X)$. If $g(X) \leq g(X^*) + \varepsilon, \forall X \in S$. It can be said that the point $X^* \in S$ is $\varepsilon -$optimal solution.

### 3.4 Fuzzy (LFP) Problem

To convert the (MOLFPP) into an (LPP), a method should be implemented such that the (LPP's) ideal solution becomes an effective one for the (MOLFPP). Variable detours, the (max-min) technique, and (α-cut) are also used in the design of our work.

The (MOLFPP) is often described as follows:

$$Maximize\left\{\frac{\tilde{C}^{num}X + \tilde{D}}{\tilde{P}^{den}X + \tilde{Q}}\right\}, \text{s.t } \tilde{A}X \leq \tilde{b},\ X \geq 0 \qquad (6)$$

Where X=($X_1$, $X_2$, … , $X_n$), the matrix is an (mxn) with fuzzy numbers, $\widetilde{b}$ and $\widetilde{a}_{ij}$ matrix of (mx1) with fuzzy numbers, (j=1, 2, 3, … , n), $(\widetilde{b}_i = 1,2,...., m)$. The equation (6) is changed by using $\alpha - cut$ as follows:

$$Maximize \quad \frac{\left[\underline{C}^{num}, \overline{C}^{num}\right]X + \left[\underline{D}, \overline{D}\right]}{\left[\underline{P}^{den}, \overline{P}^{den}\right]X + \left[\underline{Q}, \overline{Q}\right]}, \; s.t\left[\underline{A}, \overline{A}\right]X \le \left[\underline{b}, \overline{b}\right], X \ge 0 \qquad (7)$$

The following results from changing Equation (7) using operations on fuzzy intervals and numbers:

$$: Maximize \quad \overline{F}(X) = \left[\underline{F}(X), \overline{F}(X)\right] \qquad (8)$$

s.t $S = \{\underline{A}X \le \underline{b}, \overline{A}X \le \overline{b}, \overline{P}^{den}X + \overline{Q}, \underline{P}^{den}X + \underline{Q} \rangle 0, X \ge 0\}$

where

$$\underline{F}(X) = \left\{\frac{\underline{C}^{num}X + \underline{D}}{\overline{P}^{den}X + \overline{Q}}\right\}, if \; Minimize \quad \underline{C}^{num}X + \underline{D} \ge 0 \; Otherwise, \quad \underline{F}(X) = \left\{\frac{\underline{C}^{num}X + \underline{D}}{\underline{P}^{den}X + \underline{Q}}\right\}$$

And

$$\overline{F}(X) = \left\{\frac{\overline{C}^{num}X + \overline{D}}{\underline{P}^{den}X + \underline{Q}}\right\}, if \quad Maximize \; \overline{C}^{num}X + \overline{D} \ge o \quad Otherwise, \overline{F}(X) = \left\{\frac{\overline{C}^{num}X + \overline{D}}{\overline{P}^{den}X + \overline{Q}}\right\}$$

S is expected to be a regular set, as well as a non-empty, bounded feasible region. We believe

$$\underline{F}(X) = \left\{\frac{\underline{C}^{num}X + \underline{D}}{\overline{P}^{den}X + \overline{Q}}\right\}, \quad \overline{F}(X) = \left\{\frac{\overline{C}^{num}X + \overline{D}}{\underline{P}^{den}X + \underline{Q}}\right\}$$

in the rest of the paper.
Equation (8) can be expressed as follows using (Ref. [8]):

$$\underset{X \in S}{Maximize} \quad \left\{\underline{F}(X), \overline{F}(X)\right\} = \left\{\frac{\underline{C}^{num}X + \underline{D}}{\overline{P}^{den}X + \overline{Q}}, \frac{\overline{C}^{num}X + \overline{D}}{\underline{P}^{den}X + \underline{Q}}\right\} \qquad (9)$$

To convert equation (9) into non-negative numerators and positive denominators problems should be determined by the membership functions of the objectives, and it was an equivalent bi-objective problem considered in terms of the membership functions. In reality, non-negativities conditions are used to prove that this approach produces an effective solution. For this purpose, let:

$$\underset{X \in S}{Maximize}\; \underline{F}(X) = \underline{F}^{\max}, \; \underset{X \in S}{Minimize}\; \underline{F}(X) = \underline{F}^{Min}, \; \underset{X \in S}{Maximize}\; \overline{F}(X) = \overline{F}^{\max}, \; \underset{X \in S}{Minimize}\; \overline{F}(X) = \overline{F}^{Min}$$

So, the functions of membership concerning the objective functions $\underline{F}(X)$, $\overline{F}(X)$ are:

$$\underline{\mu}(x) = \frac{L^{num}X + M}{\overline{P}^{den}X + \overline{Q}}, \; \overline{\mu}(x) = \frac{N^{num}X + O}{\underline{P}^{den}X + \underline{Q}}, \quad \text{successive, where}$$

$$L = \left(\frac{\underline{C}}{\underline{F}^{\max} - \underline{F}^{\min}} - \underline{F}^{\min}\overline{P}\right), \quad M = \left(\frac{\underline{D}}{\underline{F}^{\max} - \underline{F}^{\min}} - \underline{F}^{\min}\overline{Q}\right), \text{all } X \in S \qquad (10)$$

$$N = \left(\frac{\overline{C}}{\overline{F}^{\max} - \overline{F}^{\min}} - \overline{F}^{\min}\underline{P}\right), \quad O = \left(\frac{\overline{D}}{\overline{F}^{\max} - \overline{F}^{\min}} - \overline{F}^{\min}\underline{Q}\right)$$

Since, $\underline{\mu}(x), \; \overline{\mu}(x) \in [0,1], \underline{P}^{den}X + \underline{Q}, \; \overline{P}^{den}X + \overline{Q} \rangle 0, then \; L^{num}X + M, N^{num}X + O \ge 0, forall X \in S$ According to the membership functions, equation (9) takes the following form:

$$\underset{X \in S}{Maximize}\left\{\underline{\mu}(x) = \frac{L^{num}X + M}{\overline{P}^{den}X + \overline{Q}}, \; \overline{\mu}(x) = \frac{N^{num}X + O}{\underline{P}^{den}X + \underline{Q}}\right\} \qquad (11)$$

Next using setting:

$$\lambda = Min\left\{\frac{1}{\overline{P}^{den}X + \overline{Q}}, \; \frac{1}{\underline{P}^{den}X + \underline{Q}}\right\}, \lambda X = Y, \quad \forall X \in S \qquad (12)$$

Can transform equation (10) into:

$$Maximize\left\{L^{num}Y + \lambda M, N^{num}Y + \lambda O\right\} \qquad (13)$$

s.t $\Psi = \left\{\underline{A}Y - \lambda\underline{b} \le 0, \overline{A}Y - \lambda\overline{b} \le 0, \overline{P}^{den}Y + \lambda\underline{Q} \le 1, Y, \lambda \ge 0\right\}$

assumed to ( $\Psi$ ) should be a regular set.

***Suggestion (2.7.)*** in equation (12), (λ) variable should be greater than or equal to zero

*Proof.* Let $(\hat{Y},0) \in \Psi$ then $\underline{A}\hat{Y} \le 0, \overline{A}\hat{Y} \le 0$. So, $\hat{X} \in S$ produces $\underline{A}(\hat{X} + \beta\hat{Y}) = \underline{A}\hat{X} + \beta\underline{A}\hat{Y} \le \underline{A}\hat{X} \le 0,$, $\overline{A}(\hat{X} + \beta\hat{Y}) = \overline{A}\hat{X} + \beta\overline{A}\hat{Y} \le 0, \forall\beta \ge 0$ feasibility of point of S is $\hat{X} + \beta\hat{Y}, \forall\beta \ge 0$. Therefore, S must be unlimited, this is in contrast to the fact that S is a regular set.

***Suggestion (2.8.)*** If $(\overline{Y}, \overline{\lambda}) \in \Psi$, then $\frac{\overline{Y}}{\overline{\lambda}} \in S$.

*Proof.* Where $(\overline{Y}, \overline{\lambda}) \in \Psi$, then $\overline{Y} \ge 0, \overline{\lambda}>0, \underline{A}\overline{Y} - \overline{\lambda}\underline{b}, \overline{A}\overline{Y} - \overline{\lambda}\overline{b} \le 0.$ Therefore,

$$\frac{\overline{Y}}{\overline{\lambda}} \ge 0, \underline{A}\left(\frac{\overline{Y}}{\overline{\lambda}} - \underline{b}\right) = \frac{1}{\overline{\lambda}}\left(\underline{A}\overline{Y} - \overline{\lambda}\underline{b}\right) \le 0,$$

$$\overline{A}\left(\frac{\overline{Y}}{\overline{\lambda}}\right) - \overline{b} = \frac{1}{\overline{\lambda}}\left(\overline{A}\overline{Y} - \overline{\lambda}\overline{b}\right) \le 0.$$

Let's suppose $\beta \le L^{num}Y + \lambda M, \beta \le N^{num}Y + \lambda O, \forall(Y, \lambda) \in \Psi$. It has been changed to:

$$\text{Maximize} \quad \beta \tag{14}$$

s.t

$$\Omega = \begin{Bmatrix} \underline{A}Y - \lambda\underline{b} \le 0, \overline{A}Y - \lambda\overline{b} \le 0, \overline{P}^{den}Y + \lambda\overline{Q} \le 1, \underline{P}^{den}Y + \\ \lambda\underline{Q} \le 1, \beta \le L^{num}Y + \lambda M, \beta \le N^{num}Y + \lambda O, Y, \lambda, \beta \ge 0 \end{Bmatrix},$$ here the ($\Omega$) is a regular set.

***Lemma (2.9.)*** From equation (12) had become the optimal solution is unique.

*Proof.* If it was the optimal solution is not unique based on the assumed $(Y^*, \lambda^*, \beta^*)$, that is means the constraint is active at the optimum when the $\beta \ge 0$ i.e. $\beta^* = 0.$ Let's look at the other words, if the $(Y, \lambda, \beta) \in \Omega$, so then $\beta = 0.$ so, either $L^{num}Y + \lambda M = 0$ or $N^{num}Y + \lambda O = 0, \forall(Y, \lambda, 0) \in \Omega$. Without losing the generality, let $L^{num}Y + \lambda M = 0, \forall(Y, \lambda, 0) \in \Omega$. Since $\lambda$>0, then $L^{num}Y + M = 0, \forall X \in S$; that explains $\underline{\mu}(x) = 0, \forall X \in S.$ This contradicts reducing the equation (10) to a single objective (LFP) problem.

***Theorem (2.10.)*** $X^* = \frac{Y^*}{\lambda^*}$ is the active solution for equation (10) with assume $(Y^*, \lambda^*, \beta^*)$ can be the optimal solution to equation (13).

### 3.5 Method of (FFLFP) Problem (Ref. [16])

The fact that the maximum daily needs of the linear fractional programming issue vary clearly indicates that it is an uncertain optimization problem. Thus, the quantity of each ingredient in each product will be unknown. Therefore, in order to eliminate uncertainty, we will model the fully fuzzy linear fractional programming problem in which all of the variables and all of the parameters are triangular fuzzy integers.

Let's look at a general format for equation (13) for a triangular fully fuzzy linear fractional programming problem:

Step 1. Convert into (MFFLFP) Problem as follow:

$$Max\left\{\frac{C_1X_1 + D_1}{P_1X_1 + Q_1}, \frac{C_2X_2 + D_2}{P_2X_2 + Q_2}, \frac{C_3X_3 + D_3}{P_3X_3 + Q_3}\right\} s.t \quad \widetilde{A}X \le \widetilde{b} \tag{15}$$

$$A_1X_1 \le b_1, \quad A_2X_2 \le b_2, \quad A_3X_3 \le b_3, \quad (X_1, X_2, X_3 \ge 0) \tag{16}$$

Step 2.

$$Max[(C_1X_1 + D_1), \ (C_2X_2 + D_2), \ (C_3X_3 + D_3)]$$
$$\begin{pmatrix} A_1X_1 - b_1 \le 0 \\ A_2X_2 - b_2 \le 0 \\ A_3X_3 - b_3 \le 0 \end{pmatrix}, \qquad (17)$$
$$\begin{pmatrix} P_1X_1 + Q_1 \le 1 \\ P_2X_2 + Q_2 \le 1 \\ P_3X_3 + Q_3 \le 1 \end{pmatrix},$$
$$(X_1, X_2, X_3 \ge 0)$$

Step 3.

$$\begin{aligned}
&Max(C_1X_1 + D_1) \\
&Max[(C_1X_1 + D_1) - (C_2X_2 + D_2)] \\
&Max[(C_1X_1 + D_1) + (C_3X_3 + D_3)] \\
&s.t \\
&A_1X_1 - b_1 \le 0 \\
&A_1X_1 - b_1 - A_2X_2 + b_2 \le 0 \\
&A_1X_1 - b_1 + A_3X_3 - b_3 \le 0 \\
&P_1X_1 + Q_1 \le 1 \\
&P_1X_1 + Q_1 - P_2X_2 - Q_2 \le 0 \\
&P_1X_1 + Q_1 + P_3X_3 + Q_3 \le 2 \\
&(X_1, X_2, X_3 \ge 0)
\end{aligned} \qquad (18)$$

### 3.6 Numerical Examples:

The purpose of this section is to provide an explanation of the procedures involved in the method that was utilized, as well as to take advantage of the outcomes with the intention of conducting a comparison between the new method and the concept of the approach by means of a few mathematical examples.

**Example (1):**

Take into consideration the fuzzy multi-objective linear fractional programming issue as an example of how the approach can be applicable. The work of S. K. Das and T. Mandal (2017) is the origin of this particular example.

$$\begin{aligned}
&Max = \frac{(5,1,3)X_1 + (4,1,6)X_2}{(4,6,5)X_1 + (6,3,9)X_2 + (1,2,6)} \\
&(3,2,1)X_1 + (6,4,1)X_2 \le (13,5,2) \\
&(4,1,2)X_1 + (6,5,4)X_2 \le (6,3,9) \\
&(X_1, X_2) \ge 0
\end{aligned} \qquad (19)$$

Using formula (14) to convert into (MOLFP) problem as follows:

$$\begin{aligned}
&Max\left\{\frac{5X_1 + 4X_2}{4X_1 + 6X_2 + 1}, \frac{X_1 + X_2}{6X_1 + 3X_2 + 2}, \frac{3X_1 + 6X_2}{5X_1 + 9X_2 + 6}\right\} \\
&S.T \\
&3X_1 + 6X_2 \le 13 \\
&2X_1 + 4X_2 \le 5 \\
&X_1 + X_2 \le 2 \\
&4X_1 + 6X_2 \le 6 \\
&X_1 + 5X_2 \le 3 \\
&2X_1 + 4X_2 \le 9 \\
&X_1, X_2 \ge 0
\end{aligned} \qquad (20)$$

The following is the transformation of problem (19) into an analogous multi-objective linear programming problem using formula (16):

$$
\begin{aligned}
& Max(z_1) = 5X_1 + 4X_2 \\
& Max(z_2) = X_1 + X_2 \\
& Max(z_3) = 3X_1 + 6X_2 \\
& S.T \\
& 3X_1 + 6X_2 - 13t \le 0 \\
& 2X_1 + 4X_2 - 5t \le 0 \\
& X_1 + X_2 - 2t \le 0 \\
& 4X_1 + 6X_2 - 6t \le 0 \\
& X_1 + 5X_2 - 3t \le 0 \\
& 2X_1 + 4X_2 - 9t \le 0 \\
& 4X_1 + 6X_2 + t \le 1 \\
& 6X_1 + 3X_2 + 2t \le 1 \\
& 5X_1 + 9X_2 + 6t \le 1 \\
& X_1, X_2 \ge 0
\end{aligned} \tag{21}
$$

Based on formula (17), we can be writing the problem (21) as follows:

$$
\begin{aligned}
& Max(z_1) = 5X_1 + 4X_2 \\
& Max(z_2) = 5X_1 + 4X_2 - X_1 - X_2 \\
& Max(z_3) = 5X_1 + 4X_2 + 3X_1 + 6X_2 \\
& 3X_1 + 6X_2 - 13t \le 0 \\
& 3X_1 + 6X_2 - 13t - 2X_1 - 4X_2 + 5t \le 0 \\
& 3X_1 + 6X_2 - 13t + X_1 + X_2 - 2t \le 0 \\
& 4X_1 + 6X_2 - 6t \le 0 \\
& 4X_1 + 6X_2 - 6t - X_1 - 5X_2 + 3t \le 0 \\
& 4X_1 + 6X_2 - 6t + 2X_1 + 4X_2 - 9t \le 0 \\
& 4X_1 + 6X_2 + t \le 1 \\
& 4X_1 + 6X_2 + t - 6X_1 - 3X_2 - 2t \le 0 \\
& 4X_1 + 6X_2 + t + 5X_1 + 9X_2 + 6t \le 2 \\
& X_1, X_2 \ge 0
\end{aligned} \tag{22}
$$

And we can write problem (22) as follows:

$$
\begin{aligned}
& Max(z_1) = 5X_1 + 4X_2 \\
& Max(z_2) = 4X_1 + 3X_2 \\
& Max(z_3) = 8X_1 + 10X_2 \\
& S.T \\
& 3X_1 + 6X_2 - 13t \le 0 \\
& X_1 + 2X_2 - 8t \le 0 \\
& 4X_1 + 7X_2 - 15t \le 0 \\
& 4X_1 + 6X_2 - 6t \le 0 \\
& 3X_1 + X_2 - 3t \le 0 \\
& 6X_1 + 10X_2 - 15t \le 0 \\
& 4X_1 + 6X_2 + t \le 1 \\
& -2X_1 + 3X_2 - t \le 0 \\
& 9X_1 + 15X_2 + 7t \le 2 \\
& X_1, X_2 \ge 0
\end{aligned} \tag{23}
$$

By using classical methods, the transformed linear programming problems are resolved. We get the ideal outcome.

$$
\begin{aligned}
& Z_1 = 1.0714, \quad X_1 = 1.4997, \quad X_2 = 0 \\
& Z_2 = 0.1579, \quad X_1 = 0, \quad X_2 = 0.5999 \\
& Z_3 = 0.3636, \quad X_1 = 0.8571, \quad X_2 = 0.4286
\end{aligned} \tag{24}
$$

[0.19904, 0.3406] the optimal solution

### 3.7 Describing data and building a mathematical model for (FLFPP)

The creation of the objective function, using the data from the ***Light Industries Company***, is necessary in order to build a mathematical model for the (FLFPP) of the study topic.

**Objective Function:** In our mathematical model, the fixed quantity is present in both the numerator and the denominator ($\theta_1$, $\theta_2$) respectively, and the objective function reflects maximizing the ratio between the total profits and total cost's function.

**Structural constraints for problem:** Formulating the production process constraints that achieve the objectives that the company aspires to. These constraints are as follows:

- Constraints of raw materials.
- Constraints of production capacity.
- Constraints of time (working hours).
- Constraints of order quantity.
- Constraints of non-negativity.

$$Max(\tilde{F}) = \frac{(4.731,5.125,5.591)X_1 + (5.846,6.333,6.91)X_2 + (16.154,17.5,19.091)}{(27.962,30.292,33.045)X_1 + (24.923,27,29.455)X_2 + (155.568,168.533,183.854)} \qquad (25)$$

s.t

$$\begin{array}{lll} (2.43, 2.64, 2.88)X_1+(2.43, 2.64, 2.88)X_2 & & \le (57904, 62730, 68432) \\ (1, 1, 1)X_1 & & \le (6923, 7500, 8181) \\ & (1, 1, 1)X_2 & \le (1538, 1666, 1818) \\ (1, 1, 1)X_1 & & \le (923, 1000, 1090) \\ & (1, 1, 1)X_2 & \le (153, 166, 181) \\ X_1, X_2 \ge 0 & & \end{array}$$

Using equation (1) and $\alpha = 0.8$, We get:

$$Max \quad \overline{F}(X) = \frac{[5.046,5.218]X_1 + [6.236,6.448]X_2 + [17.231,17.818]}{[29.826,30.843]X_1 + [26.585,27.491]X_2 + [165.94,171.597]} \qquad (26)$$

s.t

$$\begin{array}{lll} [2.598,2.688]X_1 + [2.598, 2.688]X_2 & & \le [61764.8, 63870.4] \\ [1, 1]X_1 & & \le [7384.6, 7636.2] \\ & [1, 1]X_2 & \le [1640.4, 1696.4] \\ [1, 1]X_1 & & \le [984.6, 1018] \\ & [1, 1]X_2 & \le [163.4, 169] \end{array}$$

The equation (8) is formulated as follows:

$$\underset{X\in S}{Max} = \left[\frac{5.046X_1 + 6.236X_2 + 17.231}{30.843X_1 + 27.491X_2 + 171.597}, \frac{5.218X_1 6.448X_2 + 17.818}{29.826X_1 26.585X_2 + 165.94}\right] \qquad (27)$$

s.t

$$\begin{array}{lll|lll} 2.598X_1 + 2.598X_2 & & \le 61764.8 & 2.688X1 + 2.688X2 & & \le 63870.4 \\ X_1 & & \le 7384.6 & X_1 & & \le 7636.2 \\ & X_2 & \le 1640.4 & & X_2 & \le 1696.4 \\ X_1 & & \le 984.6 & X_1 & & \le 1018 \\ & X_2 & \le 163.4 & & X_2 & \le 169 \end{array}$$

The equation (9) is then formulated as follows:

$$Max \quad \{\underline{F}(X), \overline{F}(X)\} = \left\{\frac{5.046Y_1 + 6.236Y_2 + 17.231}{30.843Y_1 + 27.491Y_2 + 171.597}, \frac{5.218Y_1 + 6.448Y_2 + 17.818}{29.826Y_1 + 26.585Y_2 + 165.94}\right\}$$

$$\begin{array}{ll|ll} 2.598Y_1+ 2.598Y_2 - 61764.8\lambda & \le 0 & 2.688Y_1+2.688Y_2 - 63870.4\lambda & \le 0 \\ Y_1 \quad - 7384.6\lambda & \le 0 & Y_1 \quad - 7636.2\lambda & \le 0 \\ Y_2 - 1640.4\lambda & \le 0 & Y_2 - 1696.4\lambda & \le 0 \\ Y_1 \quad - 984.6\lambda & \le 0 & Y_1 \quad - 1018\lambda & \le 0 \\ Y_2 - 163.4\lambda & \le 0 & Y_2 - 169\lambda & \le 0 \end{array} \qquad (28)$$

By using Charnes and Cooper rule, we find maxima and minima as follows:

$\underline{F}^{Max} = 0.222172, \underline{F}^{Min} = 0, \overline{F}^{Max} = 0.237746, \overline{F}^{Min} = 0$. And then define the membership functions and equation (10) are follows:

$L = 22.69693164X_1 + 28.04955722X_2 \quad M = 77.50511873$

$N = 21.9477561X_1 + 27.12133601X_2 \quad O = 74.94540401$

$$\underline{\mu}(X)=\frac{22.69693164X_1+28.04955722X_2+77.50511873}{30.843X_1+27.491X_2+171.597},$$
$$\overline{\mu}(X)=\frac{21.9477561X_1+27.12133601X_2+74.94540401}{29.826X_1+26.585X_2+165.94}$$

The formula for equation (11) is:

$$\lambda = Min \left\{ \frac{1}{30.843X_1 + 27.491X_2 + 171.597}, \frac{1}{29.826X_1 + 26.585X_2 + 165.94} \right\}, Y = \lambda X.$$

$$Maximize \quad \{22.7093Y_1+28.0648Y_2+77.5473\lambda,\quad 21.9520Y_1+27.1266Y_2+74.960\lambda\} \tag{29}$$

s.t

| | | | |
|---|---|---|---|
| $2.598Y_1+2.598Y_2 - 61764.8\lambda$ | $\le 0$ | $2.688Y_1+2.688Y_2 - 63870.4\lambda$ | $\le 0$ |
| $Y_1 - 7384.6\lambda$ | $\le 0$ | $Y_1 - 7636.2\lambda$ | $\le 0$ |
| $Y_2 - 1640.4\lambda$ | $\le 0$ | $Y_2 - 1696.4\lambda$ | $\le 0$ |
| $Y_1 - 984.6\lambda$ | $\le 0$ | $Y_1 - 1018\lambda$ | $\le 0$ |
| $Y_2 - 163.4\lambda$ | $\le 0$ | $Y_2 - 169\lambda$ | $\le 0$ |
| $30.843Y_1 + 27.491Y_2 - 171.597\lambda$ | $\le 1$ | $29.826Y_1 + 26.585Y_2 - 165.94\lambda$ | $\le 1$ |

The result of equation (13) is as follows:

$$Maximize \quad \beta \tag{30}$$

s.t $\Omega=\Psi\cup\{\beta\le 22.7093Y_1+28.0648Y_2+77.5473\lambda, \beta\le 21.9520Y_1+27.1266Y_2+74.960\lambda\}$

For the main problem, the optimal solution is: $(Y^*,\lambda^*,\beta^*)=(Y_1^*,Y_2^*,\lambda^*,\beta^*)=(0,0.03627592,0.00021465)$

$$X_1^*=\frac{0}{0.00021456}=0, X_2^*=\frac{0.03627592}{0.00021456}=169$$
$$X^*=\frac{Y^*}{\lambda^*}=(0, \quad 169)$$
$$\overline{F}(X)=\frac{6.2356(169)+17.2308}{27.491(169)+171.5972}=0.2223208$$
$$\underline{F}(X)=\frac{6.4484(169)+17.8182}{26.5846(169)+165.94}=0.2377464$$

---

We will look at example (1) by T. Mandal and S. K. Das (2017). A similar multi-objective linear fractional programming (MOLFP) problem was created in this instance from the (FFLFP) problem. Afterward, (MOLFP) was transformed using a mathematical programming technique into a similar multi-objective linear programming problem. by using a new method here, we use the same example and compare the results.

$$\begin{aligned} Max &= \frac{(5,1,3)X_1 + (4,1,6)X_2}{(4,6,5)X_1 + (6,3,9)X_2 + (1,2,6)} \\ &(3,2,1)X_1 + (6,4,1)X_2 \le (13,5,2) \\ &(4,1,2)X_1 + (6,5,4)X_2 \le (6,3,9) \\ &(X_1, X_2) \ge 0 \end{aligned} \tag{31}$$

Using equation (1) and $\alpha = 0.8$, We get:

$$\begin{aligned} Max &= \frac{[1.8,1.4]X_1+[1.6,2]X_2}{[5.6,5.8]X_1+[3.6,4.2]X_2+[1.8,2.8]} \\ &[2.2,1.8]X_1+[4.4,3.4]X_2\le[6.6,4.4] \\ &[1.6,1.2]X_1+[5.2,4.8]X_2\le[3.6,4.2] \end{aligned} \tag{32}$$

Equation (8) is formulated as follows:

$$\begin{aligned} \underset{X\in S}{Max} &= \left[\frac{1.4X_1+1.6X_2}{5.8X_1+4.2X_2+2.8}, \frac{1.8X_1+2X_2}{5.6X_1+3.6X_2+1.8}\right] \\ S.T & \\ &1.8X_1+3.4X_2\le 4.4 \quad \downarrow 2.2X_1+4.4X_2\le 6.6 \\ &1.2X_1+4.8X_2\le 3.6 \quad \downarrow 1.6X_1+5.2X_2\le 4.2 \end{aligned} \tag{33}$$

The equation (9) is then formulated as follows:

$$\underset{X\in S}{Max}\{\underline{F}(X),\overline{F}(X)\}=\left\{\frac{1.4X_1+1.6X_2}{5.8X_1+4.2X_2+2.8},\frac{1.8X_1+2X_2}{5.6X_1+3.6X_2+1.8}\right\} \quad (34)$$

$S.T$

$$1.8X_1+3.4X_2-4.4\lambda\le 0 \quad \downarrow 2.2X_1+4.4X_2-6.6\lambda\le 0$$
$$1.2X_1+4.8X_2-3.6\lambda\le 0 \quad \downarrow 1.6X_1+5.2X_2-4.2\lambda\le 0$$

By using ***Charnes and Cooper rule***, We find maxima and minima as follows:
$\underline{F}^{Max}=0.2071, \underline{F}^{Min}=0, \overline{F}^{Max}=0.3431, \overline{F}^{Min}=0$. And then define the membership functions and equation (10) are following:

$$L=\frac{1.4X_1+1.6X_2}{0.2071}=6.76X_1+7.7257X_2 \quad , \quad M=0$$
$$N=\frac{1.8X_1+2X_2}{0.3431}=5.246X_1+5.829X_2 \quad , \quad O=0$$
$$\underline{\mu}(X)=\frac{6.76X_1+7.7257X_2}{5.8X_1+4.2X_2+2.8} \quad , \quad \overline{\mu}(X)=\frac{5.246X_1+5.829X_2}{5.6X_1+3.6X_2+1.8}$$

The equation (11) is then formulated as follows:

$$\lambda=Min\left\{\frac{1}{5.8X_1+4.2X_2+2.8},\frac{1}{5.6X_1+3.6X_2+1.8}\right\}, \quad Y=\lambda X$$
$$Max\{6.76Y_1+7.7257Y_2 \quad , \quad 5.246Y_1+5.829Y_2\}$$
$$5.8Y_1+4.2Y_2+2.8\lambda=1 \quad , \quad 5.6Y_1+3.6Y_2+1.8\lambda=1 \quad (35)$$
$$1.8Y_1+3.4Y_2-4.4\lambda\le 0 \quad , \quad 2.2Y_1+4.4Y_2-6.6\lambda\le 0$$
$$1.2Y_1+4.8Y_2-3.6\lambda\le 0 \quad , \quad 1.6Y_1+5.2Y_2-4.2\lambda\le 0$$

Solving by (LP) package and the optimal solutions to the problem are:
$Z=1.0001 \quad Y_1=0 \quad Y_2=0.1716 \quad \lambda=0.2124$

$$Y=\lambda X\rightarrow \quad X_1=\frac{0}{0.2124}=0 \quad X_2=\frac{00.1716}{0.2124}=0.8079$$
$$\underline{F}(X)=\frac{1.4(0)+1.6(0.8079)}{5.8(0)+4.2(0.8079)+2.8}=0.2087$$
$$\overline{F}(X)=\frac{1.8(0)+2(0.8079)}{5.6(0)+3.6(0.8079)+1.8}=0.3431$$

**3.7.1 Discussion:** $\underline{\mu}(x)$ and $\overline{\mu}(x)$ have an average of (1). As we can see, our novel approach effectively tackles problem (18) since the average of the member ship functions unity, achieving an average membership function equal one suggests a high level of effectiveness in the methodology. This could be a crucial indication of the reliability and strength of our novel approach. It is evident that the [16] method's solution yielded a value for the function between (0.19904, 0.3406), but our new technique produced values for the function between (0.2087, 0.3431). Accordingly, the improvement methodology has been achieved.

## 4. Conclusions

The fuzzy problem was eventually transformed into an (LPP). It has been demonstrated that the (LPP)-the generated solution is an ϵ-optimal solution to the core problem. Our methodology was built using the membership function, the max-min technique, the concept of (α-cut), and variable transformations. This article covers the (LFPP) with any type of fuzzy numbers.

According to information from the Iraqi Light Industries Company, this bi-objective linear fractional programming problem with fuzzy coefficients was resolved, yielding $X^*$, which provides a unique optimal solution for the primary fuzzy problem.

In a short, we draw the conclusion that the method we utilized is flexible for addressing the (MOLFPP) with fuzzy coefficients and multi-objective (MOLFPP).

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