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On Triplea-g-Transformation and Its Properties

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ABSTRACT: In this paper, we defined new triple transformation, which is called the fractional triple g-transformation of the order $\alpha, 0 < \alpha \leq 1$ for fractional of differentiable functions. This transformation is generalized to double g −transformation. Which has the following form;

$\mathrm{Tg}_\alpha \left(u(\xi,\tau,\mu) = p(s)\int_0^\infty \int_0^\infty \int_0^\infty E_\alpha\left(-\left(q_1(s)\xi + q_2(s)\tau + q_3(s)\mu\right)^\alpha (d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha\right.\right.$

Keywords: Triple transformation, Mittag-Leffler function, T-transformation, Fractional convolution problem, Integral transform.

1. INTRODUCTION

Transforms are powerful tools to solve many engineering and technological problems. It also has wide importance in many fields, including science, engineering, astronomy, and others. The important part of the integrative transformation is the nucleus of this transformation because through the nucleus we can distinguish the type of this transformation. In [1] Jafari introduced a general integral transformation and called it the g transformation. He also studied the properties of this transformation and its applications in differential equations, The mittag-leffler function is an important function and is considered a generalization of the [2], [3] exponential function In this work, we used the mittag-leffler function as an alternative to the exponential function in g- transform. Also in this paper we studied the properties of the fractional triple g transform and its inverse and its applications in fractional derivatives because most solutions of fractional differential equations are related to the mitttag-leffler function. In [4] the triple g transform and its properties were used to solve fractional order partial differential equations based on the Riemann-Liouville fractional derivative. And the caputo fractional derivative, we also presented many theories and examples related to the subject of the paper

2. FRACTIONAL TRIPLE g_α-TRANSFORMATION

DEFINITION 2.1 [5, 6]:

Let u(ξ, τ) be piecewise function. and its continuous where $\xi, \tau > 0$ and $\xi \in (0, \infty)$. then the Double g-transformation Dg(u(ξ, τ)) is defined by the following integral:

$$Dg\left(u\left(\xi,\tau\right)\right) = P_1 P_2 \int_0^\infty \int_0^\infty e^{-q_1\xi - q_2\tau} u\left(\xi,\tau\right) dxdt \tag{1}$$

such that the integral is convergent for some" $q_1(s)$, $q_2(s)$ are positive functions, and

$\|\,\mathrm{Dg}(u(\xi,\tau))\| \leq \frac{P_1 P_2 L}{k - q_1 q_2}, \quad |u(\xi,\tau)| \leq Le^{k(\xi+\tau)}$

DEFINITION 2.2 [7]:

Let $u(\xi,\tau,\mu)$ be a function of three variables where $\xi,\tau,\mu \in [0,\infty)$, then the Triple g-transformation of $u(\xi,\tau,\mu)$ is defined as;

$$Tg_3\left(u\left(\xi,\tau,\mu\right)\right) = p\left(s\right)\int_0^\infty\int_0^\infty\int_0^\infty e^{-q_1(s)\xi-q_2(s)\tau-q_3(s)\mu}u\left(\xi,\tau,\mu\right)d\xi d\tau d\mu \qquad , p(s) = \quad p_1p_2p_3 \tag{2}$$

Such that $\xi,\tau,\mu > 0$ and s is positiveI constant and $\sup\left(\frac{u(\xi,\tau,\mu)}{e^{a\xi+b\tau+c\mu}}\right| < 0 \qquad , a,b,c \in R$

The inverse of $Tg_3 - transform$ It is expressed by the following relationship

$$u\left(\xi,\tau,\mu\right) = \frac{1}{2\pi i}\int_{\lambda-i\pi}^{\lambda-i\pi} e^{-q_1(s)\xi-q_2(s)\tau-q_3(s)\mu}U\left(s\right)ds$$

DEFINITION 2.3 [8]:

The r-gamma function is given by the formula:

$$\Gamma_\alpha\left(\omega\right) = \frac{1}{\Gamma(\alpha+1)}\int_0^\infty t^{(\omega-1)^\alpha}E_\alpha(t)^\alpha(dt)^\alpha \qquad , Re\left(\omega\right) > 0 \quad r > 1 \tag{3}$$

In a special case when $\alpha = 1$ "we get the classical gamma function"" Γ_ω

$$\Gamma_{(\omega)} = \int_0^\infty t^{\omega-1}e^{-t}dx \qquad Re\left(\omega\right) > 0$$

DEFINITION 2.4:

Let $u(\xi,\tau,\mu)$ be a function of three variables where ξ,τ,μ >0, then we define a fractional Triple g_α-transformationl by the following from:

$$Tg_\alpha(u\left(\xi,\tau,\mu\right) = p\left(s\right)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-\left(q_1\left(s\right)\xi + q_2\left(s\right)\tau + q_3\left(s\right)\mu\right)^\alpha\ (d\xi)^\alpha(d\tau)^\alpha d\left(\mu\right)^\alpha \tag{4}$$

Where $E_\alpha\left(\omega\right)$ is mittag-liffler function $E_\alpha\left(\omega\right) = \sum_{i=0}^\infty \frac{\omega^i}{\Gamma(\alpha i+1)}$,where p(s)>0 and $q_1\left(s\right) \quad , \quad q_2\left(s\right)\ q_3\left(s\right) > 0$

$$, p\left(s\right) = p_1p_2p_3$$

REMARK 2.5:

The definition (2,4) can be written in another form using the properties of the mitttag-leffler function as follow

$$Tg_\alpha(u\left(\xi,\tau,\mu\right) = p\left(s\right)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-\left(q_1\left(s\right)\xi\right)^\alpha).E_\alpha(-(q_2\left(s\right)\tau)^\alpha).E_\alpha\left(-(q_3\left(s\right)\mu)^\alpha\right)\ (d\xi)^\alpha(d\tau)^\alpha d\left(\mu\right)^\alpha \tag{5}$$

PROPOSITION 2.6:

If $u_1(\xi,\tau,\mu)$ and $u_2(\xi,\tau,\mu)$ are a functions of three vairables ξ,τ,μ ,then

$$Tg_\alpha\left(a_1u_1\left(\xi,\tau,\mu\right) + a_2u_2\left(\xi,\tau,\mu\right) + a_3u_3\left(\xi,\tau,\mu\right)\right) = a_1Tg_\alpha\left(\xi,\tau,\mu\right) + a_2Tg_\alpha\left(\xi,\tau,\mu\right) + a_3Tg_\alpha\left(\xi,\tau,\mu\right) \tag{6}$$

Proof:

The proof is performed from Definition (2.4)

EXAMPLES 2.7:

$$1)\ Tg_\alpha\left(1\right) = p\left(s\right)\left(\left(\int_0^\infty E_\alpha(q_1)^\alpha\left(1\right)(d\xi)^\alpha\right).\ \left(\int_0^\infty E_\alpha(q_1\tau)^\alpha(d\tau)^\alpha\right).\left(\int_0^\infty E_\alpha(q_1\mu)^\alpha(d\mu)^\alpha\right)\right]$$

$$= p\left(s\right)[\left(\frac{\Gamma_\alpha\left(1\right)\Gamma\left(\alpha+1\right)}{q_1^\alpha}\right).\left(\frac{\Gamma_\alpha\left(1\right)\Gamma\left(\alpha+1\right)}{q_1^\alpha}\right).\left(\frac{\Gamma_\alpha\left(1\right)\Gamma\left(\alpha+1\right)}{q_1^\alpha}\right)$$

$$= \frac{p\left(s\right)}{q_1^\alpha q_2^\alpha q_3^\alpha}\Gamma_\alpha^3\left(1\right)\Gamma^3\left(\alpha+1\right)$$

$$2)\quad Tg_\alpha\left(\xi^n\right) = p\left(s\right)\left(\left(\int_0^\infty E_\alpha(q_1\xi)^\alpha\left(\xi^n\right)(d\xi)^\alpha\right).\ \left(\int_0^\infty E_\alpha(q_1\tau)^\alpha(d\tau)^\alpha\right).\left(\int_0^\infty E_\alpha(q_1\mu)^\alpha(d\mu)^\alpha\right)\right]$$

$$= p(s)\left[\left(\frac{\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)}{q_1^{\alpha(n+1)}}\right).\left(\frac{\Gamma(1)\Gamma(\alpha+1)}{q_2^{\alpha}}\right).\left(\frac{\Gamma(1)\Gamma(\alpha+1)}{q_3^{\alpha}}\right)\right]$$

$$= \frac{p(s)\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma^2(1)\Gamma^3(\alpha+1)}{q_1^{\alpha(n+1)}q_2^{\alpha}q_3^{\alpha}}$$

3) $$Tg_\alpha\left(\xi^n\tau^m\mu^k\right) = p(s)\left[\left(\int_0^\infty E_\alpha(q_1\xi)^\alpha(\xi^n)(d\xi)^\alpha\right).\left(\int_0^\infty E_\alpha(q_1\tau)^\alpha(\tau^m)(d\tau)^\alpha\right).\left(\int_0^\infty E_\alpha(q_1\mu)^\alpha\left(\mu^k\right)(d\mu)^\alpha\right)\right]$$

$$= p(s)\left[\left(\frac{\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma(\alpha+1)}{q_1^{\alpha(n+1)}}\right).\left(\frac{\Gamma_\alpha\left(\frac{m}{\alpha}+1\right)\Gamma(\alpha+1)}{q_2^{\alpha(m+1)}}\right).\left(\frac{\Gamma_\alpha\left(\frac{k}{\alpha}+1\right)\Gamma(\alpha+1)}{q_3^{\alpha(k+1)}}\right)\right]$$

$$= \frac{p(s)\Gamma_\alpha\left(\frac{n}{\alpha}+1\right)\Gamma_\alpha\left(\frac{m}{\alpha}+1\right)\Gamma_\alpha\left(\frac{k}{\alpha}+1\right)\Gamma^3(\alpha+1)}{q_1^{\alpha(n+1)}q_2^{\alpha(m+1)}q_3^{\alpha(k+1)}}$$

PROPOSITION 2.8;

$$\mathrm{Tg}_\alpha(\mathrm{u}(\mathrm{a}\xi,\mathrm{b}\tau,\mathrm{c}\mu)) = \frac{\mathrm{p(s)}}{\mathrm{a}^\alpha\mathrm{b}^\alpha\mathrm{c}^\alpha}\mathrm{U}_\alpha\left(\frac{\mathrm{q}_1}{\mathrm{a}},\frac{\mathrm{q}_2}{\mathrm{b}}\cdot\frac{\mathrm{q}_3}{\mathrm{c}}\right), \text{Where a, b, c are constants.} \quad (7)$$

Proof:

$$Tg_\alpha(u(a\xi,b\tau,c\mu)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1\xi+q_2\tau+q_3\mu)^\alpha)\,u(a\xi,b\tau,c\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

$$let\ \lambda = a\xi \quad , \quad \gamma = b\tau \quad , \beta = c\mu$$

$$Tg_\alpha(u(a\xi,b\tau,c\mu)) = \frac{p(s)}{a^\alpha b^\alpha c^\alpha}\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-\left(\frac{q_1u}{a}+\frac{q_2v}{b}+\frac{q_3w}{c}\right)^\alpha)\,u(\lambda,\gamma,\beta)\,(d\lambda)^\alpha(d\gamma)^\alpha d(\beta)^\alpha$$

$$= \frac{p(s)}{a^\alpha b^\alpha c^\alpha}U_\alpha\left(\frac{q_1}{a},\frac{q_2}{b}.\frac{q_3}{c}\right)$$

PROPOSITION 2.9:

$$Tg_\alpha(E_\alpha(-(a\xi+b\tau+c\mu)^\alpha)u(\xi,\tau,\mu)) = p(s)\,U_\alpha(q_1+a,q_2+b,q_2+c) \quad (8)$$

Proof;

$$Tg_\alpha(E_\alpha(-(a\xi+b\tau+c\mu)^\alpha)u(\xi,\tau,\mu)) = p\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1\xi+q_2\tau+q_3\mu)^\alpha)\,E_\alpha(-(a\xi+b\tau+c\mu))^\alpha u(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

$$By\ using.the\ formla \qquad E_\alpha(L(\xi+\tau+\mu)^\alpha = E_\alpha(L\xi^\alpha)E_\alpha(L\tau^\alpha)E_\alpha(L(\mu^\alpha)$$

Then we have.

$$Tg_\alpha(E_\alpha(-(a\xi+b\tau+c\mu)^\alpha)u(\xi,\tau,\mu)) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(a+q_1)\xi+(b+q_2)\tau+(c+q_3)\mu)^\alpha)\,u(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

$$Tg_\alpha\left(E_\alpha(-(a\xi+b\tau+c\mu)^\alpha)u(\xi,\tau,\mu)\right)=p(s)\,U_\alpha(q_1+a,q_2+b,q_2+c)$$

PROPOSITION 2.10:

$$Tg_\alpha\left(\xi^\alpha\tau^\alpha\mu^\alpha u(\xi,\tau,\mu)\right)=\frac{p(s)\partial^{3\alpha}}{\partial q_1^\alpha\partial q_2^\alpha\partial q_3^\alpha}Tg_\alpha\left(u(\xi,\tau,\mu)\right) \tag{9}$$

Proof:

$$Tg_\alpha\left(\xi^\alpha\tau^\alpha\mu^\alpha u(\xi,\tau,\mu)\right)=p(s)\int_0^\infty\int_0^\infty\int_0^\infty \xi^\alpha E_\alpha\left(-(q_3\xi)^\alpha\right).\tau^\alpha E_\alpha\left(-(q_3\tau)^\alpha\right).\mu^\alpha E_\alpha\left(-(q_3\mu)^\alpha\right)\; u(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

By using.the equality $\quad D_s^\alpha\left(E_\alpha\left(-st^\alpha\right)\right)=-t^\alpha E_\alpha(-st^\alpha).$

Then we obtain.

$$Tg_\alpha\left(\xi^\alpha\tau^\alpha\mu^\alpha u(\xi,\tau,\mu)\right)=p(s)\int_0^\infty\int_0^\infty\int_0^\infty \frac{\partial^\alpha}{\partial q_1^\alpha}E_\alpha\left(-(q_3\xi)^\alpha\right).\frac{\partial^\alpha}{\partial q_2^\alpha}E_\alpha\left(-(q_3\tau)^\alpha\right).\frac{\partial^\alpha}{\partial q_3^\alpha}E_\alpha\left(-(q_3\mu)^\alpha\right)\; u(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

$$=p(s)\int_0^\infty\int_0^\infty\int_0^\infty \frac{\partial^{3\alpha}}{\partial q_1^\alpha\partial q_2^\alpha\partial q_3^\alpha}E_\alpha\left(-(q_3\xi)^\alpha\right).E_\alpha\left(-(q_3\tau^\alpha\right).E_\alpha\left(-(q_3\mu)^\alpha\right)\; u(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

Therefor

$$Tg_\alpha\left(\xi^\alpha\tau^\alpha\mu^\alpha u(\xi,\tau,\mu)\right)=\frac{p(s)\partial^{3\alpha}}{\partial q_1^\alpha\partial q_2^\alpha\partial q_3^\alpha}Tg_\alpha\left(u(\xi,\tau,\mu)\right)$$

DEFINITION 2.11 [9]:

Let u, v are the functions of three variable such that , $\tau,\mu>0$, then the fractional triple convolution is defined as.follows

$$\left(u(\xi,\tau,\mu)***_\alpha v(\xi,\tau,\mu)\right)=\int_0^\xi\int_0^\tau\int_0^\mu u(\xi-\lambda,\ \tau-\gamma,\ \mu-\beta)v(\lambda,\gamma,\beta)\,(d\xi)^\alpha(d\tau)^\alpha(d\mu)^\alpha \tag{10}$$

THEOREM 2.12:

Let u and v be a function then

$$Tg_\alpha\left(u***_\alpha v\right)(\xi,\tau,\mu)=\frac{1}{p(s)}Tg_\alpha\left(u(\xi,\tau,\mu)\right).Tg_\alpha\left(v(\xi,\tau,\mu)\right) \tag{11}$$

Proof:

$$Tg_\alpha\left(u***_\alpha v\right)(\xi,\tau,\mu)=p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1\xi+q_2\tau+q_3\mu)^\alpha)\;\left(u***_\alpha v\right)(\xi,\tau,\mu)\,(d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

$$=p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1\xi)^\alpha).E_\alpha(-(q_2\tau)^\alpha)\,.\,E_\alpha(-(q_3\mu)^\alpha)\;\times\Big[\int_0^\xi\int_0^\tau\int_0^\mu u(\xi-\lambda,\ \tau-\gamma,\ \mu\quad-\beta)$$

$v(\lambda,\gamma,\beta)(d\lambda)^\alpha(d\gamma)^\alpha(d\beta)^\alpha](d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$

Let $n=\xi-\lambda\quad,\quad m=\tau-\gamma\quad,\quad k=\mu-\beta\qquad$ *and we take limit from* 0 *to* ∞

$$=p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-q_1^\alpha(n+\lambda)^\alpha).\;E_\alpha(-q_2^\alpha(m+\gamma)^\alpha.E_\alpha(-q_1^\alpha(k+\beta)^\alpha$$

$$\times\left(\int_0^\infty\int_0^\infty\int_0^\infty u(n,\ m,\ k)v(\lambda,\gamma,\beta)(d\lambda)^\alpha(d\gamma)^\alpha(d\beta)^\alpha](dn)^\alpha(dm)^\alpha(dk)^\alpha\right]$$

$$= p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-q_1^\alpha n^\alpha\right).\,E_\alpha\left(-q_2^\alpha m^\alpha\right).E_\alpha\left(-q_1^\alpha k^\alpha\right)u(n,m,k)(dn)^\alpha(dm)^\alpha d(k)^\alpha \times \int_0^\infty\int_0^\infty\int_0^\infty E_\alpha\left(-q_1^\alpha\lambda^\alpha\right).\,E_\alpha\left(-q_2^\alpha\gamma^\alpha\right).E_\alpha\left(-q_1^\alpha\beta^\alpha\right)v(\lambda,\gamma,\beta)(d\lambda)^\alpha(d\gamma)^\alpha d(\beta)^\alpha$$

Thus

$$Tg_\alpha(u**\ast_\alpha v)(x,t,z) = \frac{1}{p(s)}Tg_\alpha(u(\xi,\tau,\mu)).Tg_\alpha(v(\xi,\tau,\mu))$$

DEFINITION 2.13:

The fractional delta function of three variables $\delta_\alpha(\xi-n,\ \tau-m,\ \mu-k)$,$\alpha\quad 0<\alpha\le 1$ is defined as follows that.

$$\int_R\int_R\int_R u(\xi,\tau,\mu)\ \delta_\alpha(\xi-n,\ \tau-m,\ \mu-k)(d\xi)^\alpha(d\tau)^\alpha(d\mu)^\alpha = p(s)\alpha^3 u(n,m,k) \tag{12}$$

EXAMPLE 2.14;

We taking $\delta_\alpha(\xi-n,\ \tau-m,\ \mu-k)$ then.

$$Tg_\alpha((\xi-n,\ \tau-m,\ \mu-k)\} = p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1\xi+q_2\tau+q_3\mu)^\alpha)\delta_\alpha(\xi-n,\ \tau-m,\ \mu-k)(d\xi)^\alpha(d\tau)^\alpha(d\mu)^\alpha$$

$$= p(s)\alpha^3E_\alpha(-(q_1\xi+q_2\tau+q_3\mu)^\alpha)$$

THEOREM 2.15;

Let $U_\alpha g_3(s)$ be the triple $g_\alpha - transform\ of\ u(\xi,\tau,\mu)$ which define in following formula :

$$Tg_\alpha(u(\xi,\tau,\mu) = U_\alpha g_3(q_1q_2q_3) = p(s)\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1(s)\xi+q_2(s)\tau+q_3(s)\mu)^\alpha\ (d\xi)^\alpha(d\tau)^\alpha d(\mu)^\alpha$$

then the invers formula is defined as;

$$Tg_\alpha^{-1}\left(U_{\alpha g}(s)\right) = u(\xi,\tau,\mu) = \frac{1}{p(s)(m_\alpha)^{3\alpha}}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}E_\alpha((q_1(s)\xi+q_2(s)\tau+q_3(s)\mu)^\alpha)U_{\alpha g}(s)(dq_1)^\alpha(dq_2)^\alpha(dq_3)^\alpha \tag{13}$$

Proof :

$$u(\xi,\tau,\mu) = \frac{1}{p(s)(m_\alpha)^{3\alpha}}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}E_\alpha(q_1\xi)^\alpha E_\alpha(q_2\tau)^\alpha E_\alpha(q_3\mu)^\alpha(dq_1)^\alpha(dq_2)^\alpha(dq_3)^\alpha\times\int_0^\infty\int_0^\infty\int_0^\infty E_\alpha(-(q_1(s)\,\xi+q_2(s)\tau+q_3(s)\mu^\alpha\ u(\lambda,\gamma.\beta)(d\lambda)^\alpha(d\gamma)^\alpha(d\beta)^\alpha$$

$$= \frac{1}{p(s)(m_\alpha)^{3\alpha}}\int_0^\infty\int_0^\infty\int_0^\infty u(\lambda,\gamma.\beta)(d\lambda)^\alpha(d\gamma)^\alpha(d\beta)^\alpha\times\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty}E_\alpha(-q_1^\alpha(\xi-\lambda)^\alpha).\,\mathrm{E}_\alpha(-q_2^\alpha(\tau-\gamma)^\alpha.E_\alpha(-q_1^\alpha(z-\beta)^\alpha(dq_1)^\alpha(dq_2)^\alpha(dq_3)^\alpha$$

$$= \frac{1}{p(s)(m_\alpha)^{3\alpha}}\int_0^\infty\int_0^\infty\int_0^\infty\frac{(m_\alpha)^{3\alpha}}{\alpha^3}u(\lambda,\gamma.\beta)\ \delta_\alpha(\lambda-\xi,\ \gamma-\tau,\ \beta-\mu)(d\lambda)^\alpha(d\gamma)^\alpha(d\beta)^\alpha$$

$$= \frac{1}{p(s)\alpha^3}\int_0^\infty\int_0^\infty\int_0^\infty u(\lambda,\gamma.\beta)\ \delta_\alpha(\lambda-\xi,\ \gamma-\tau,\ \beta-\mu)(d\lambda)^\alpha(d\gamma)^\alpha(d\mu)^\alpha$$

$$= u(\xi,\tau,\mu)$$

3. FRACTIONAL TRIPLE g_α-TRANSFORMATIO FOR FRACTIONAL PARTIAL DERIVATIVES

In this section we apply the fractional triple transformation to find some partial derivatives. We will explain it through the following theorems;

THEOREM 3.1:

Let $u = (\xi, \tau, \mu)$ be a function of three variables such that,$\xi, \tau, \mu \in (0, \infty)$, $\alpha. \in R^{+}l$ then.

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial \xi^\alpha} f(\xi, \tau, \mu)\right) = q_1^\alpha F_\alpha(p, q_1 q_2, q_3) - \Gamma(1+\alpha) g_\alpha(f(0, \tau, \mu), p_2, q_2) \tag{14}$$

Proof:

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial \xi^\alpha} f(\xi, \tau, \mu)\right) = p(s)\int_0^\infty \int_0^\infty \int_0^\infty E_\alpha(-q_1^\alpha \xi^\alpha) E_\alpha(-q_2^\alpha \tau^\alpha) E_\alpha(-q_3^\alpha \mu^\alpha) \frac{\partial^\alpha}{\partial \xi^\alpha} f(\xi, \tau, \mu)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha$$

$$= p_1 \int_0^\infty \left(E_\alpha\left(-q_1^\alpha \xi^\alpha\right) \frac{\partial^\alpha}{\partial \xi^\alpha} f(\xi, \tau, \mu)(d\xi)^\alpha\right] p_2 p_3 \int_0^\infty \int_0^\infty E_\alpha\left(-q_2^\alpha \tau^\alpha\right) E_\alpha(-q_2^\alpha \mu^\alpha)(d\tau)^\alpha (d\mu)^\alpha$$

By using fractional integration by part formula in the inner integral then we get:

$$= p_2 p_3 \int_0^\infty \int_0^\infty E_\alpha\left(-q_2^\alpha \tau^\alpha\right) E_\alpha(-q_2^\alpha \mu^\alpha) \left[\Gamma(1+\alpha) E_\alpha\left(-q_1^\alpha \xi^\alpha\right) f(\xi, \tau, \mu)\right]_{\xi=0}^\infty - \int_0^\infty \left(E_\alpha\left(-q_1^\alpha \xi^\alpha\right)\right.$$

$$\partial^\alpha \overline{\partial \xi^\alpha f(\xi,\tau,\mu)(d\xi)^\alpha (d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha}$$

$$= -\Gamma(1+\alpha) p_2 p_3 \int_0^\infty \int_0^\infty E_\alpha\left(-q_2^\alpha \tau^\alpha\right) E_\alpha(-q_3^\alpha \mu^\alpha)\, u(0, \tau, \mu)(d\tau)^\alpha (d\mu)^\alpha + q_1^\alpha p_1 p_2 p_3 \int_0^\infty \int_0^\infty \int_0^\infty E_\alpha\left(-q_1^\alpha \xi^\alpha\right)$$

$$E_\alpha\left(q_2^\alpha \tau^\alpha\right) u(\xi, \tau, \mu)(d\tau)^\alpha (d\xi)^\alpha (d\mu)^\alpha$$

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial \xi^\alpha} f(\xi, \tau, \mu)\right) = q_1^\alpha U_\alpha(p, q_1 q_2, q_3) - \Gamma(1+\alpha) D g_\alpha(u(0, \tau, \mu), p_2, q_2)$$

THEOREM 3.2:

Let $u = (\xi, \tau, \mu)$ be a function of three variables such that ,$\xi, \tau, \mu \in (0, \infty)$, $\alpha. \in R^{+}l$ then

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial \tau^\alpha} u(\xi, \tau, \mu)\right) = q_2^\alpha U_\alpha(p, q_1 q_2, q_3) - \Gamma(1+\alpha) g_\alpha(u(\xi, 0, \mu), p_1, q_1) \tag{15}$$

Proof:

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial \tau^\alpha} f(\xi, \tau, \mu)\right) = p(s)\int_0^\infty \int_0^\infty \int_0^\infty E_\alpha(-q_1^\alpha \xi^\alpha) E_\alpha(-q_2^\alpha \tau^\alpha) E_\alpha(-q_3^\alpha \mu^\alpha) \frac{\partial^\alpha}{\partial \xi^\alpha} u(\xi, \tau, \mu)(d\xi)^\alpha (d\tau)^\alpha (d\mu)^\alpha$$

$$= p_1 p_3 \int_0^\infty \int_0^\infty E_\alpha\left(-q_1^\alpha \tau^\alpha\right) E_\alpha(-q_3^\alpha \mu^\alpha)(d\xi)^\alpha (d\mu)^\alpha p_2 \int_0^\infty \left(E_\alpha\left(-q_1^\alpha \tau^\alpha\right) \frac{\partial^\alpha}{\partial \tau^\alpha} u(\xi, \tau, \mu)(d\tau)^\alpha\right]$$

By using fractional integration by part formula in the inner integral then we get :

$$= p_1 p_3 \int_0^\infty \int_0^\infty E_\alpha\left(-q_1^\alpha \xi^\alpha\right) E_\alpha(-q_3^\alpha \mu^\alpha)[\ [\Gamma(1+\alpha)$$

$$E_\alpha\left(-q_2^\alpha \tau^\alpha\right) u(\xi, \tau, \mu)]_{\xi=0}^\infty - p_2 \int_0^\infty \left(\frac{\partial^\alpha}{\partial \tau^\alpha} E_\alpha\left(-q_1^\alpha \tau^\alpha\right)\right) u(\xi, \tau, \mu)(d\xi)^\alpha](d\tau)^\alpha (d\mu)^\alpha$$

$$= -\Gamma(1+\alpha)p_1p_3\int_0^{\infty}\int_0^{\infty}E_\alpha\left(-q_1^\alpha\xi^\alpha\right)E_\alpha(-q_3^\alpha\mu^\alpha)u\,(0,\tau,\mu)\,(d\xi)^\alpha(d\mu)^\alpha + q_1^\alpha p_1p_2p_3\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}E_\alpha\left(-q_1^\alpha\xi^\alpha\right)E_\alpha\left(q_2^\alpha\tau^\alpha\right)$$

$E_\alpha(-q_3^\alpha\mu^\alpha)u(\xi,\tau,\mu)(d\tau)^\alpha(d\xi)^\alpha(d\mu)^\alpha$

$$Tg_\alpha\left(\frac{\partial^\alpha}{\partial\tau^\alpha}u\,(\xi,\tau,\mu)\right) = q_1^\alpha U_\alpha\,(p,q_1q_2,q_3) - \Gamma(1+\alpha)Dg_\alpha(U(0,\tau,\mu),p_2,q_2)$$

THEOREM 3.3:

Let u be a function $\xi,\tau,\mu\in(0,\infty)$, $\alpha\in R^+$,then

$Tg_\alpha\left(\frac{\partial^{2\alpha}}{\partial\xi^\alpha\partial\tau^\alpha}u\,(\xi,\tau,\mu)\right) = (\alpha!)^2u\,(0,0,\mu) - (\alpha!)\,q_2^\alpha U_\alpha\,(0,q_2,q_3) - (\alpha!)\,q_1^\alpha U_\alpha\,(q_1,0,q_3) + q_1^\alpha q_2^\alpha q_3^\alpha U_\alpha\,(\xi,\tau,\mu)$

Proof:

The proof is complete by using the Theorem (3.1) and Theorem (3.2)

4. CONCLUSION

In this article, we have covered a new definition of the fractional triple g-transform and its inverse. as we discussed some of the characteristics of this transformation, while studying some theorems and examples. In addition, we found the fractional triple transformation for fractional partial order derivatives.

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CONFLICTS OF INTEREST

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REFERENCES

[1] H. Jafari, “A new general integral transform for solving integral equations,” *J. Adv. Res*, vol. 32, pp. 133–138, 2021.
[2] A. Apelblat, “Differentiation of the mittag-leffier functions with respect to parameters in the laplace transform approach,” *Mathematics*, vol. 8, no. 5, 2020.
[3] H. J. Haubold, “Mittag-Leffler Functions and Their Applications,” *J. Appl. Math*, vol. 1, pp. 1–51, 2011.
[4] R. J. Al-Owaidi and M. H. Geem, “Solving Fractional Partial Differential Equations by Triple g-Transformation,” *J. Al-Qadisiyah for Comput. Sci. Math*, vol. 15, no. 1, pp. 86–91, 2023.
[5] A. Abbood and A. M. H. Geem, “Double α-g-Transformation and Its Properties,” *J. Al-Qadisiyah for Comput. Sci. Math*, vol. 14, no. 3, pp. 33–41, 2022.
[6] M. Meddahi, H. Jafari, and X. J. Yang, “Towards new general double integral transform and its applications to differential equations,” *Math. Methods Appl. Sci*, vol. 45, no. 4, pp. 1916–1933, 2022.
[7] R, J. Hadi, and A. M. H. Geem, “On Triple g-Transformation and Its Properties,” *J. Al-Qadisiyah for Comput. Sci. Math*, vol. 14, no. 4, pp. 174–183.
[8] M. A. Khan and S. Ahmed, “On some properties of the generalized Mittag-Leffler function,” *SpringerPlus*, vol. 2, pp. 337–337, 2013.
[9] G. D. Medina, “Fractional Laplace Transform and Fractional Calculus,” *J. Int. Math. Forum*, pp. 991–999, 2017.